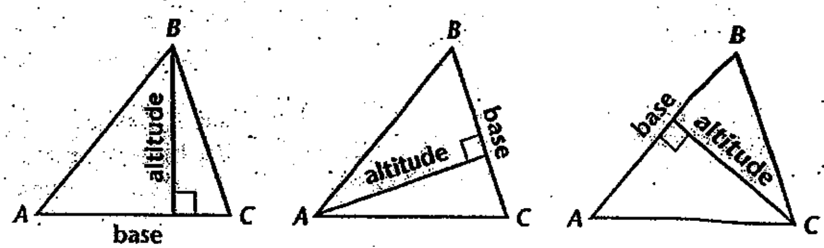
NAME ________________________________ CLASS ____________ DATE ____________

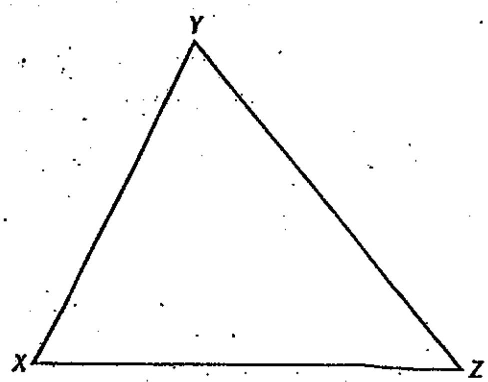
Enrichment

1.5 Altitudes and the Orthocenter

Any side of a triangle can be designated as a **base**. The perpendicular segment from the vertex opposite a base to a line containing that base is an **altitude** of the triangle. Every triangle has three altitudes.



Trace $\triangle XYZ$ at right onto a piece of paper.



1. Fold the paper so that $\overline{XZ}$ folds onto itself and the fold line passes through point Y. Mark the point where the fold intersects $\overline{XZ}$ and label it P. Draw $\overline{YP}$. This is the altitude from vertex Y to base $\overline{XZ}$.

2. Fold the paper to construct the altitude from vertex X to base $\overline{ZY}$. Label it $\overline{XQ}$. Then construct the altitude from vertex Z to base $\overline{YX}$. Label it $\overline{ZR}$.

3. If you have carefully completed Exercises 1 and 2, the altitudes should be concurrent. The point of concurrency of the lines that contain the altitudes is called the **orthocenter** of the triangle. Is it possible for the orthocenter to be in the exterior of the triangle? Can it be on the triangle? Explain.

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4. **a.** On a sheet of paper, draw a large triangle with sides of different lengths. Make three copies of the triangle by tracing it onto three other sheets of paper. Locate one of the following on each triangle: the orthocenter, O; the incenter, I; the circumcenter, C; the centroid, D.

   b. Make another copy of the triangle. Trace the orthocenter, incenter, circumcenter, and centroid onto this copy, being careful to label each correctly.

   c. If you have carefully constructed the points of concurrency, three of them should be collinear. Which three are they? (The line that contains these points is called the **Euler line**.)

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   d. There is a special relationship among the lengths of the segments that join the collinear points. Using your drawing, make a conjecture about this relationship.

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   e. Test your conjecture from part **d** by repeating parts **a** and **b** with a different triangle.

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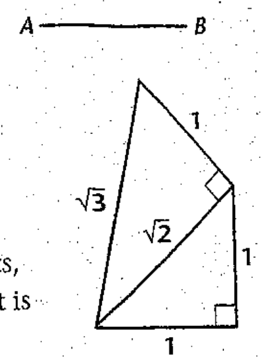
5.4 Constructing Segments with Irrational Lengths

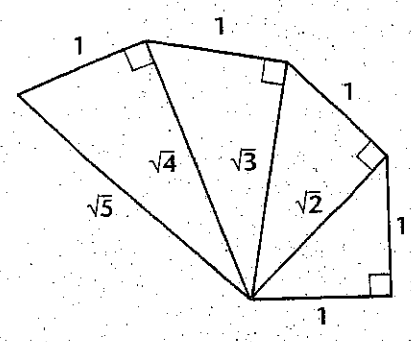


At right is shown a segment, $\overline{AB}$. Consider its length to be 1 unit.

Suppose that you construct a right triangle with legs of length 1 unit, as shown at right. Then, by the Pythagorean Theorem, the length of the hypotenuse is $\sqrt{2}$ units. If you then construct an adjacent right triangle as shown, with legs of length $\sqrt{2}$ units and 1 unit, then the length of its hypotenuse is $\sqrt{3}$ units.

Continuing this process, you can construct segments of length $\sqrt{4}$ units, $\sqrt{5}$ units, $\sqrt{6}$ units, and so on. The resulting construction is called the *wheel of Theodorus*. It is named for the Greek philosopher Theodorus of Cyrene (465–398, B.C.E.), who is know for his contributions to the understanding of irrational numbers. Before his work, $\sqrt{2}$ was the only known irrational number. Theodorus showed that $\sqrt{3}$, $\sqrt{5}$, $\sqrt{6}$, $\sqrt{7}$, $\sqrt{8}$, $\sqrt{10}$, $\sqrt{11}$, $\sqrt{12}$, $\sqrt{13}$, $\sqrt{14}$, $\sqrt{15}$, and $\sqrt{17}$ are also irrational.

1. In the figure below, the wheel of Theodorus has been constructed through $\sqrt{5}$. Using compass and straightedge, continue the construction through $\sqrt{17}$.



2. To construct a segment of length $\sqrt{24}$ units, you could continue constructing the wheel through $\sqrt{24}$. Describe an alternative method. (Hint: How can you use algebra to simplify $\sqrt{24}$?)

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