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12.1 Quantified Statements

Many logical statements contain one of the words *all*, *some*, or *no*. These words are called **quantifiers** because they indicate *how many*, or *quantity*. A statement that contains a quantifier is called a **quantified statement.**

In a quantified statement, *no* has the same meaning as in everyday usage. *All* is equivalent to *each* or *every*. Although people generally use *some* to mean "at least two, but not all," in logic it means "at least one, and possibly all."

Give the true value of each statement.

1. All rectangles are squares. ________
2. All squares are rectangles. ________
3. Some rectangle is a square. ________
4. Some squares are rectangles. ________
5. No rectangles are squares. ________
6. No square is a rectangle. ________
7. All rectangles are not squares. ________
8. Some squares are not rectangles. ________

Given a logical statement, *p*, the statement *not p* is called its **negation.** You can negate many statements simply by negating the verb. For instance, the negation of *A square is a triangle* is the statement *A square is not a triangle.* To negate a quantified statement, however, you must follow the rules in the chart at the right.

To negate:	Do this:
an "all" statement	Replace *all* with *some* and negate the verb.
a "some" statement	Replace *some* with *all* and negate the verb.
a "no" statement	Replace *no* with *some*.

Negate each statement. Give the truth value of the statement and its negation.

9. All rectangles are quadrilaterals.

____

____

10. Some trapezoids are isosceles trapezoids.

____

____

11. *No parallelogram is a rhombus.*

____

____

12. *Some rectangle is not a parallelogram.*

____

____

On a separate sheet of paper, explain why each argument is *not* a correct application of the Law of Indirect Reasoning. Then correct the argument.

13. If *ABCDE* is a quadrilateral, then some quadrilaterals have five sides.
Some quadrilaterals do not have five sides.
Therefore, *ABCDE* is not a quadrilateral.

14. If all rhombuses are squares, then all rhombuses have four right angles.
All rhombuses do not have four right angles.
Therefore, all rhombuses are not squares.

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12.2 A Further Investigation of Equivalence

If two statements are logically equivalent, then they have the same truth value. The study of logic often requires you to demonstrate whether two statements are logically equivalent. You can usually do this by completing a truth table.

1. Show that p OR (p AND q) is equivalent to p AND (p OR q).

p	q	p AND q	p OR q	p OR (p AND q)	p AND (p OR q)	~p and ~q	p or ~q [redacted]
T	T						
T	F						
F	T						
F	F						

2. Show that $\sim(p$ OR $q)$ is *not* equivalent to $\sim p$ OR $\sim q$.

p	q	$\sim p$	$\sim q$	p OR q	$\sim(p$ OR $q)$	$\sim p$ OR $\sim q$	~(p or ~q)	#3	#4	#5
T	T									
T	F									
F	T									
F	F									

Match each expression in Set I with an equivalent expression in Set II. Record your answers in the blanks at right.

Set I	Set II	
3. $\sim q$ OR $\sim(p$ OR $\sim q)$	**a.** p	3. ________
4. $\sim p$ AND (p OR $\sim q$)	**b.** q	4. ________
5. $\sim q$ OR ($\sim p$ AND $\sim q$)	**c.** $\sim p$	5. ________
6. (p AND q) OR (p AND $\sim q$)	**d.** $\sim q$	6. ________
7. (p AND q) OR $\sim(p$ OR $\sim q)$	**e.** p OR q	7. ________
8. $\sim(p$ AND $\sim q)$ AND $\sim(p$ AND $q)$	**f.** p AND q	8. ________
9. (p OR q) OR ((p OR q) AND q)	**g.** $\sim p$ OR $\sim q$	9. ________
10. ((p OR q) AND p) AND ((p OR q) AND q)	**h.** $\sim p$ AND $\sim q$	10. ________

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12.3 Using Logical Chains to Draw Conclusions

Recall that, in Chapter 2, you learned the If-Then Transitive Property. This property allows you to create a *logical chain* of conditionals. When you are given a set of premises, you can often use a logical chain as part of a valid argument to arrive at a desired conclusion.

IF-THEN TRANSITIVE PROPERTY	
When you are given:	*You can conclude:*
If p then q	If p then r
If q then r	

At right is a set of given premises and a desired conclusion. Write a reason that justifies each statement in the following argument.

Premises: $p \Rightarrow q$
$\sim p \Rightarrow r$
Conclusion: $\sim q \Rightarrow r$

1. $p \Rightarrow q$ ______________________
2. $\sim q \Rightarrow \sim p$ ______________________
3. $\sim p \Rightarrow r$ ______________________
4. $\sim q \Rightarrow r$ ______________________

Write a valid argument to show that the stated conclusion follows from the given premises. Be sure to write the reason that justifies each statement.

5. Premises: $p \Rightarrow q$
$\sim r \Rightarrow \sim q$
Conclusion: $p \Rightarrow r$

6. Premises: $\sim p \Rightarrow q$
$p \Rightarrow \sim r$
$\sim q$
Conclusion: $\sim r$

Write a conclusion that follows from the given set of premises.

7. $p \Rightarrow q$
$q \Rightarrow \sim r$
$\sim s \Rightarrow r$
____

8. $\sim p \Rightarrow q$
$r \Rightarrow \sim q$
r
____

9. $p \Rightarrow \sim q$
q OR $\sim r$
r
____

10. $p \Rightarrow q$
$\sim(q \text{ AND } r)$
r
____

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2.4 Properties of Inequality

Just as with the properties of equality, you can use the *properties of inequality* to draw conclusions about geometric figures. Some fundamental properties of inequality are listed at right.

PROPERTIES OF INEQUALITY

Let a, b, and c be real numbers.

Addition Property	If $a > b$, then $a + c > b + c$.
Subtraction Property	If $a > b$, then $a - c > b - c$.
Multiplication Property	If $a > b$ and $c > 0$, then $ac > bc$.
	If $a > b$ and $c < 0$, then $ac < bc$.
Division Property	If $a > b$ and $c > 0$, then $a \div c > b \div c$.
	If $a > b$ and $c < 0$, then $a \div c < b \div c$.
Transitive Property	If $a > b$ and $b > c$, then $a > c$.

In Exercises 1–4, name the property of inequality that justifies each conclusion.

1. Given: $JK > MN$
Conclusion: $JK + 5 > MN + 5$

____

2. Given: $m\angle 1 > m\angle 2$
Conclusion: $-m\angle 1 < -m\angle 2$

____

3. Given: $UV + XY > WZ + XY$
Conclusion: $UV > WZ$

____

4. Given: $m\angle A > m\angle Z$; $m\angle B < m\angle Z$
Conclusion: $m\angle A > m\angle B$

____

5. You are given that point M is the midpoint of $\overline{XY}$, point N is the midpoint of $\overline{WZ}$, and $\overline{XY}$ is longer than $\overline{WZ}$. Which is longer: $\overline{XM}$ or $\overline{WN}$? Use the properties of inequality to justify your answer.

____

____

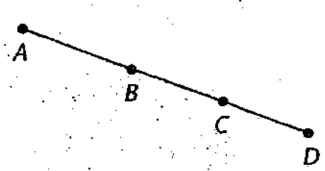
6. You are given that $\angle ABC$ is larger than $\angle XYZ$. Which is larger, the complement of $\angle ABC$ or the complement of $\angle XYZ$? Justify your answer by using the properties of inequality.

____

____

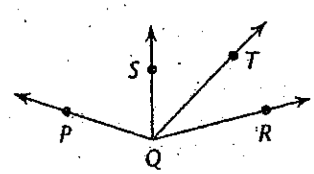
On a separate sheet of paper, write a proof of each theorem.

7.



Given: A, B, C, and D are collinear.
$AB > CD$
Prove: $AC > BD$

8.



Given: S and T are in the interior of $\angle PQR$.
$m\angle PQT > m\angle SQR$
Prove: $m\angle PQS > m\angle TQR$