

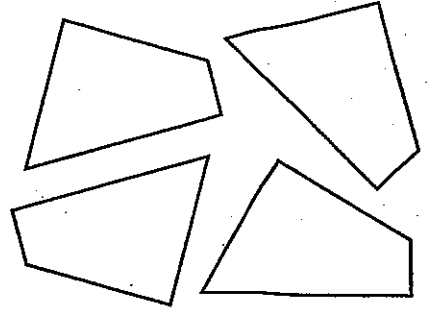


Enrichment

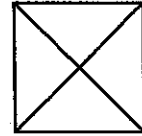
4.1 Dissections

In the exercises on this page, you will explore a fascinating branch of mathematics that is called *dissection theory*.

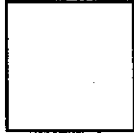
- Carefully trace the four figures at right onto a sheet of paper. Cut them out. Arrange the figures so that together they form a square. Sketch the arrangement in the blank space at right.



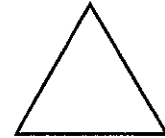
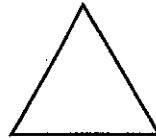
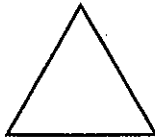
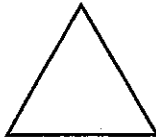
When you **dissect** a geometric figure, you cut it into two or more parts. The puzzle pieces in Exercise 1 were formed by dissecting a square into four congruent polygons. The figures at right show three other dissections.



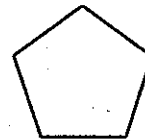
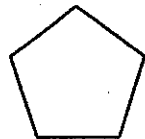
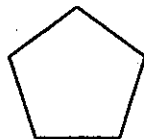
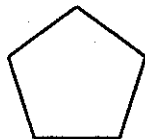
- Show four additional ways to dissect a square into four congruent polygons. (The polygons may be either convex or concave.)



- Show four ways to dissect an equilateral triangle into *three* congruent polygons.

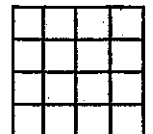


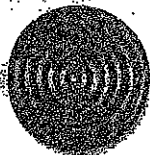
- Show four ways to dissect a regular pentagon into *five* congruent polygons.



- Describe a general technique for dissecting any regular n -gon into n congruent polygons.

- The figure at right is a 4-by-4 grid of squares. Making cuts only along the grid lines, find all possible ways to dissect the grid into *two* congruent parts. Sketch your dissections on a separate sheet of paper.



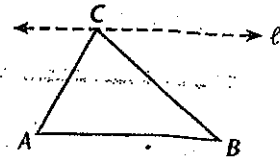


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3.5 Auxiliary Figures

An **auxiliary figure** is a line, ray, segment, or other figure that is added to a given figure, usually as an aid to a calculation or proof. You may use an auxiliary figure only if it is uniquely **determined**. That is, there must be exactly one figure that satisfies the stated conditions. There must also be a postulate, theorem, or definition that justifies the existence of the auxiliary figure.

In proving the Triangle Sum Theorem, auxiliary line ℓ is drawn through vertex C of $\triangle ABC$ so that it is parallel to \overline{AB} , the side opposite C .



If you do not place enough conditions on an auxiliary figure, it is said to be **undetermined**. If there are too many conditions, the figure is **overdetermined**.

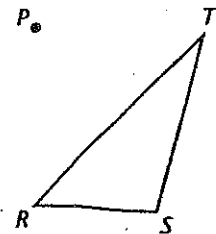
You are given the figure at right below. State whether each instruction describes an auxiliary figure that is **uniquely determined**, **undetermined**, or **overdetermined**. Justify your answer.

1. Draw line ℓ through T . _____

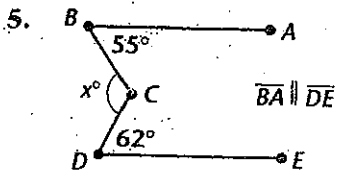
2. Draw \overleftrightarrow{PT} . _____

3. Draw line m through T parallel to \overline{RS} . _____

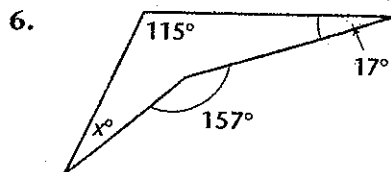
4. Draw \overleftrightarrow{PT} parallel to \overline{RS} . _____



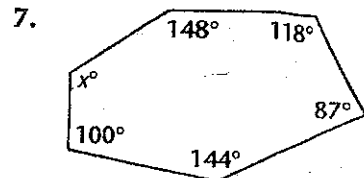
Draw one or more auxiliary figures that will help you find the value of x . Then find the value of x .



$x =$ _____

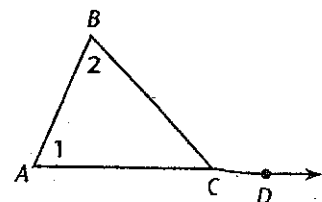


$x =$ _____



$x =$ _____

8. Often there is more than one way to prove a theorem. For instance, it is possible to prove the Exterior Angle Theorem by inserting an auxiliary figure. Use the figure at right to show how this might be done. Write a complete proof on a separate sheet of paper. (Hint: Think about the angles formed when parallel lines are cut by a transversal.)





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4.3 Congruent Triangles on the Coordinate Plane

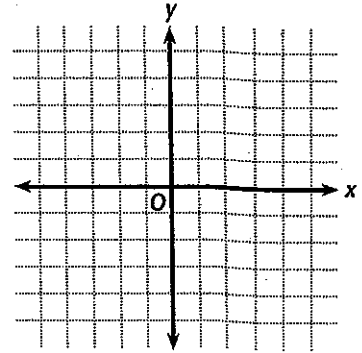
When proving that two triangles are congruent, you must sometimes use your knowledge of lines on the coordinate plane.

Graph each set of lines. Then: a. Identify two congruent triangles that are formed by the lines. b. Write a paragraph proof to justify your statement in part a.

1. $y = 5$ $y = -x + 1$
 $y = -4$ $y = -3x + 2$

a. _____

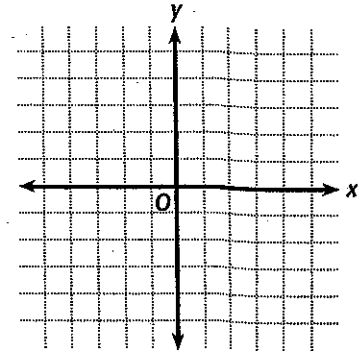
 b. _____



2. $y = 3$ $y = 2x + 1$
 $x = -1$ $y = -2x - 3$

a. _____

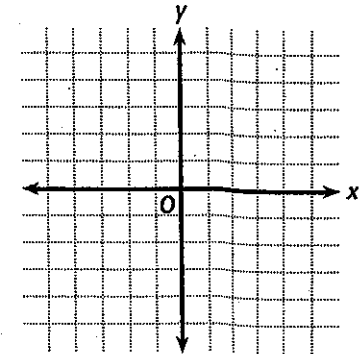
 b. _____



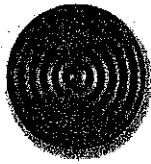
3. $y = -4$ $y = x - 2$
 $x = -2$ $y = -x$

a. _____

 b. _____



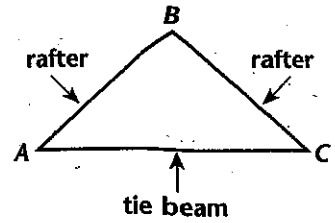
4. On a separate sheet of paper, write equations for a different set of lines that meet to form congruent triangles. Identify the congruent triangles, and write a proof to justify the congruence.



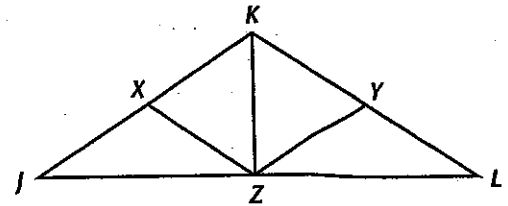
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4.4 Isosceles Triangles and Roof Trusses

The wooden or metal framework that supports a roof is called a roof *truss*. The simplest type of roof truss has the shape of an isosceles triangle, as depicted by $\triangle ABC$ at right. In the diagram, the legs of the triangle, \overline{AB} and \overline{CB} , represent sloping beams that are called *rafters*. The base, \overline{AC} , represents the *tie beam* that "ties together" the rafters. However, large roofs require trusses with designs that are more complex than this.



For example, a *king-post truss* is pictured at right. The king post, \overline{KZ} , is a median of $\triangle JKL$, and it provides support for the rafters. Additional support for the rafters comes from *struts* \overline{ZX} and \overline{ZY} , which are medians of $\triangle JKZ$ and $\triangle LKZ$, respectively. The outer triangle, $\triangle JKL$, is an isosceles triangle with base \overline{JL} .

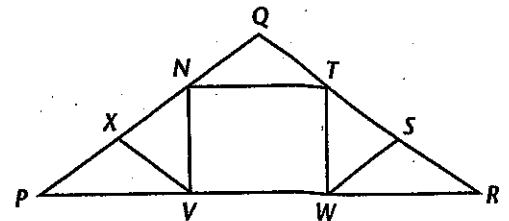


1. Refer to the diagram of the king-post truss. Write a flowchart proof to show that $\angle JKZ \cong \angle LKZ$.

2. Explain why it must be true that $\angle XJZ \cong \angle XZJ$. (Hint: How is \overline{ZX} related to $\triangle JKL$?)

3. Given that $m\angle KJZ = 33^\circ$, find the measures of all the other angles between the rafters, beams, and struts of the king-post truss. Label the angle measures directly on the figure.

In the *queen-post truss* pictured at right, congruent queen posts \overline{NV} and \overline{TW} are positioned so that they are perpendicular to tie beam \overline{PR} and so that $\overline{PV} \cong \overline{VW} \cong \overline{WR}$. Struts \overline{MV} , \overline{SW} , and \overline{NT} intersect the rafters so that $\overline{PM} \cong \overline{MN} \cong \overline{NQ}$ and $\overline{RS} \cong \overline{ST} \cong \overline{TQ}$. The outer triangle, $\triangle PQR$, is an isosceles triangle with base \overline{PR} .



4. Using the information about the queen-post truss given above, prove each statement on a separate sheet of paper. Use any form of proof that you want.

- a. $\triangle QNT$ is isosceles.
- b. $\triangle MPV$ is isosceles.
- c. $\triangle VMN$ is isosceles.

5. Given that $m\angle NQT = 110^\circ$, find the measures of all the other angles between the rafters, beams, and struts of the queen-post truss. Label the angle measures directly on the figure.