NAME _______________ CLASS _______________ DATE _______________

Enrichment
4.8 Other Inequalities in a Triangle

The Triangle Inequality Theorem describes a relationship among the lengths of the sides of a triangle. The following two theorems relate the lengths of the sides to the measures of the angles.

UNEQUAL SIDES THEOREM	UNEQUAL ANGLES THEOREM
If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.	If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.

1. In $\triangle XYZ$, $XY = 9.3$, $YZ = 7.6$, and $XZ = 8.05$. Name the largest and smallest angles of $\triangle XYZ$.

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2. In $\triangle JKL$, $m\angle J = 62°$ and $m\angle K = 57°$. Name the longest and shortest sides of $\triangle JKL$.

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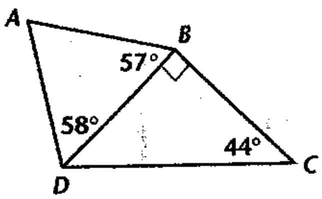
In each figure, list the segments in order from longest to shortest.

3. 

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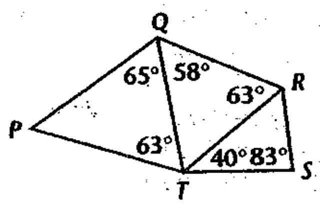
4. 

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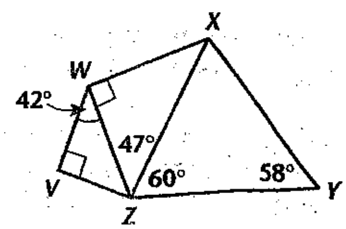
5. 

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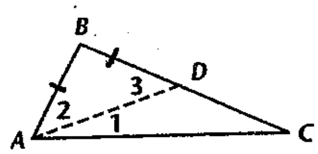
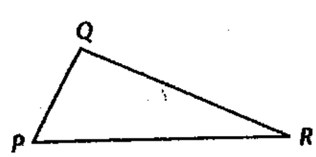
On a separate sheet of paper, write a proof of each theorem.

6. *Unequal Sides Theorem*
Given: $\triangle ABC$ with $BC > AB$
Prove: $m\angle BAC > m\angle C$



Plan for proof: Locate point D on $\overline{BC}$ such that $BD = BA$. Draw $\overline{AD}$. Explain why $m\angle BAC > m\angle 3$, $m\angle 3 > m\angle C$, and so $m\angle BAC > m\angle C$.

7. *Unequal Angles Theorem*
Given: $\triangle PQR$ with $m\angle P > m\angle R$
Prove: $QR > QP$



Plan for proof: The three possible relationships between QR and QP are $QR = QP$, $QR < QP$, and $QR > QP$. Show that the first two relationships listed are impossible.

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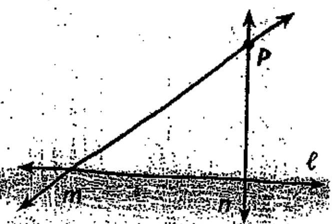
Enrichment
12.4 Indirect Proofs of Theorems About Perpendicular Lines

The method of indirect proof makes it possible to prove the following fundamental theorem about perpendicular lines.

Through a point outside a line, there is exactly one line perpendicular to the given line.

Complete the following indirect proof of the above theorem.



Given: Lines m and n each pass through point P, not on line ℓ.
$m \perp \ell$
$n \perp \ell$
Prove: Lines m and n are the same line.

Assume that lines m and n are **1.** ______________________.

Since lines m and n each pass through point P, they have a point in common. Therefore, by the definition of **2.** ______________________, they are intersecting lines. However, since lines m and n are each perpendicular to line ℓ, they are **3.** ______________________ to each other, because **4.** ____.

Thus, the assumption that lines m and n are different lines leads to the following contradiction: lines m and n are **5.** ______________ and lines m and n are **6.** ______________.
Therefore, the assumption must be **7.** ______________________. The conclusion is that
8. ____.

9. Describe a different plan for an indirect proof of the above theorem.

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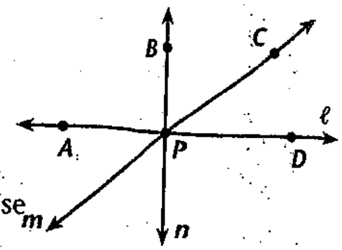
10. A second fundamental theorem about perpendicular lines is:

Through a point on a line, there is exactly one line perpendicular to the given line.



Using the figure at right, write an indirect proof of this theorem. (Hint: Use the Linear Pair Property in relation to $\angle APB$ and $\angle BPD$. Use the Angle Addition Postulate in relation to $\angle BPC$ and $\angle CPD$.)