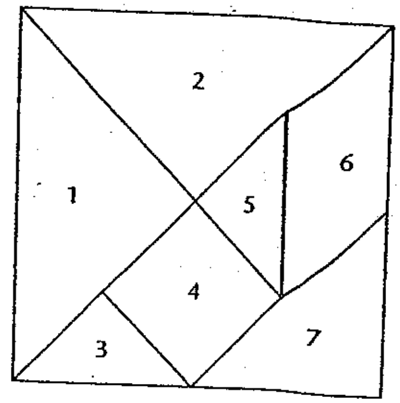
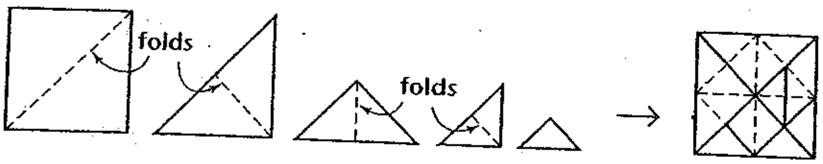
NAME ______________________ CLASS ____________ DATE ____________

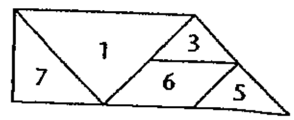
# Enrichment
## 3.2 Quadrilaterals and Tangrams

The figure at right shows a Chinese puzzle called the *tangram*. No one is certain how old the puzzle is, but it was unquestionably one of the most popular puzzles of the nineteenth century. The seven puzzle pieces are called *tans*. To make your own tans, fold a large square sheet of paper in half four times as shown below. Unfold the paper, draw the segments shown, then cut along the segments.

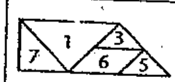


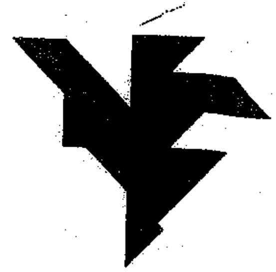

Some or all of the tans can be arranged to form special types of quadrilaterals. For instance, the figure at right shows how tans 1, 3, 5, 6, and 7 can be arranged to form a trapezoid.




**Complete the table below by arranging the given number of tans to form each figure. If it is not possible to make the figure, enter an × in the table.**

| | Number of tans | Square | Rectangle (not a square) | Isosceles trapezoid | Trapezoid (not isosceles) | Rhombus (not a square) | Parallelogram (not a rhombus or rectangle) |
|---|---|---|---|---|---|---|---|
| **1.** | 2 | | | | | | |
| **2.** | 3 | | | | | | |
| **3.** | 4 | | | | | | |
| **4.** | 5 | | | |  | | |
| **5.** | 6 | | | | | | |
| **6.** | 7 | | | | | | |

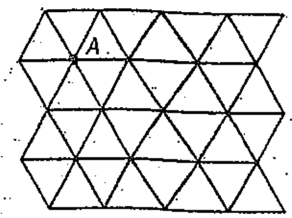
**7.** **a.** Tangram puzzles arise from the countless silhouettes of people, animals, and objects that can be formed by arrangements of the tans. Find an arrangement of the tans that forms the silhouette of a bird, as shown at right. Sketch your answer on a separate sheet of paper.



**b.** Create an original tangram puzzle. Sketch the silhouette on a piece of paper. Trade puzzles with a friend and solve each other's puzzles.

# Enrichment
## 3.6 Regular and Semiregular Tessellations

A **tessellation** is a pattern of congruent figures that completely covers a plane without gaps or overlaps. A tessellation that consists of exactly one type of regular polygon, with each polygon congruent to all the others, is called a **regular tessellation**. Any point where the polygons share a common vertex is a **vertex point** of the tessellation. The figure at right, for instance, shows a regular tessellation of equilateral triangles with one vertex point labeled *A*.




**For Exercises 1–3, refer to the regular tessellation above.**

1. What is the measure of each interior angle of each equilateral triangle? ____________

2. How many equilateral triangles meet at each vertex point? ____________

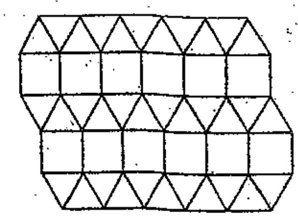
3. What is the sum of the measures of all the angles at each vertex point? ____________

4. Based on your answers to Exercises 1–3, explain why it is not possible to make a regular tessellation that consists of regular pentagons.

   ______________________________________________

   ______________________________________________

A **semiregular tessellation** is a tessellation of two or more types of regular polygons *with congruent sides* that has the same arrangement of polygons at each vertex point. It is important to specify the arrangement. The figure at right, for instance, is a semiregular tessellation of equilateral triangles and squares. Using 3 to represent a triangle and 4 to represent a square, you specify the arrangement at each vertex point as 3-3-3-4-4.



5. Show calculations to verify that the sum of the measures of the angles at each vertex point of the 3-3-3-4-4 tessellation is 360°.

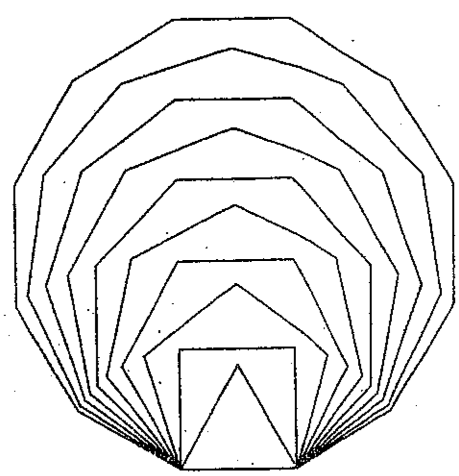
   ______________________________________________

6. Explain why the tessellation with code 3-3-3-4-4 is different from the tessellation with code 3-3-4-3-4.

   ______________________________________________

   ______________________________________________

7. Trace and cut out several copies of the regular polygons in the figure at right. Working with these copies, find all possible regular and semiregular tessellations. On a separate sheet of paper, sketch each tessellation, record its code, and show calculations to verify that the sum of the angle measures at each vertex is 360°.



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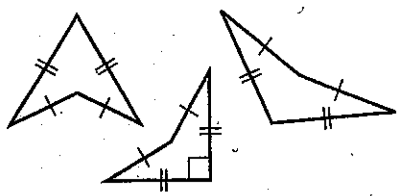
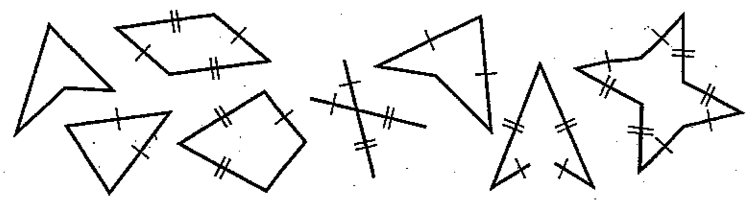
# Enrichment
## 4.5 *Defining and Analyzing Darts*

On this page you will investigate a special type of figure called a *dart.*

These figures are **darts.**



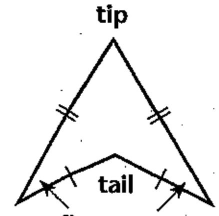
These figures are *not* darts.



**1.** Write a definition of the term *dart.*

______________________________________________

______________________________________________




**2.** Before you can discuss darts and make conjectures about them, you need to define terms that describe their special features. These terms are labeled on the dart at right. Complete each of the following definitions.

**a.** The **tip** of a dart is the vertex of the angle formed by the ______________________ pair of congruent sides.

**b.** The **tail** of a dart is the union of the two ______________________ congruent sides.

**c.** A **fin angle** of a dart is an angle formed by a pair of ______________________ sides.

**3.** Write four conjectures about the special properties of darts.

______________________________________________

______________________________________________

______________________________________________

______________________________________________

**4.** Choose two of the conjectures that you made in Exercise 3. On a separate sheet of paper, write a proof of each conjecture. Then state your conjecture as a theorem and give each theorem a title.

**5.** Is it possible for a dart to have four congruent sides? Explain.

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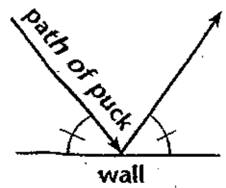
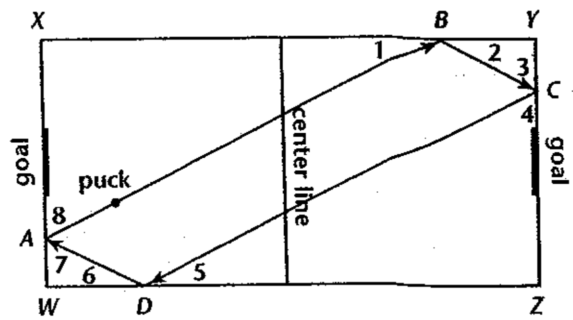
# Enrichment

## 4.6 Finding Special Quadrilaterals on an Air Hockey Table




In the game of air hockey, a puck glides on a thin layer of air above a rectangular table which is 4 feet wide and 8 feet long. The object of the game is to hit the puck into your opponent's goal. The movement of the puck is confined to the table by walls at the edges of the table. Because of a physical principle, the *law of reflection,* the angle at which the puck bounces off a wall is congruent to the angle at which it strikes the wall.

**In the figure at right, a puck has been hit in such a way that it bounces off each wall of the table exactly once. Complete the following proof that the path shown is a parallelogram.**




Given: *WXYZ* is a rectangle; $m\angle 1 = m\angle 2$; $m\angle 3 = m\angle 4$; $m\angle 5 = m\angle 6$; $m\angle 7 = m\angle 8$

Prove: *ABCD* is a parallelogram.

By the **1.** ______________ Theorem, $m\angle 1 + m\angle 8 + m\angle X = 180°$ and $m\angle 6 + m\angle 7 + m\angle W = 180°$. So $m\angle 1 = 180° - m\angle 8 - m\angle X$ and $m\angle 6 = 180° - m\angle 7 - m\angle W$. It is given that $m\angle 8 =$ **2.** __________. Because *WXYZ* is a rectangle, $m\angle X = m\angle W =$ **3.** __________. So, using properties of equality, $m\angle 1 = m\angle 6$. By similar reasoning, $m\angle 2 =$ **4.** __________. You know that $m\angle 1 + m\angle ABC + m\angle 2 =$ **5.** __________ and that $m\angle 5 + m\angle CDA + m\angle 6 =$ **6.** __________. So $m\angle ABC = 180° - m\angle 1 - m\angle 2$ and $m\angle CDA = 180° - m\angle 6 - m\angle 5$. Using properties of equality, it follows that $m\angle ABC =$ **7.** __________. By similar reasoning, $m\angle BCD =$ **8.** __________. You know that $m\angle ABC + m\angle BCD + m\angle CDA + m\angle DAB =$ **9.** __________. So, using properties of equality, $m\angle ABC + m\angle BCD + m\angle ABC + m\angle BCD = 360°$, or $2(m\angle ABC) + 2(m\angle BCD) = 360°$. It follows that $2(m\angle ABC + m\angle BCD) = 360°$, or $m\angle ABC + m\angle BCD = 180°$. This means that $\angle ABC$ and $\angle BCD$ are a pair of **10.** ______________ angles. Similarly, $\angle BCD$ and $\angle CDA$ are a pair of **11.** ______________ angles. So $\overline{AB} \parallel \overline{DC}$ and $\overline{BC} \parallel \overline{AD}$ by the **12.** ______________ Theorem.

Therefore, *ABCD* is a parallelogram by the **13.** ______________ of a parallelogram.

**Can the path of a puck have the given shape? Write an explanation on a separate sheet of paper.**

**14.** rhombus **15.** rectangle **16.** kite