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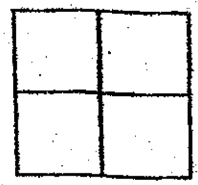
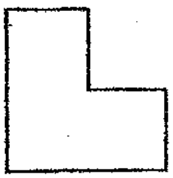
Enrichment
8.2 Rep-tiles

When you dissect a geometric figure, you cut it into two or more parts. If the parts are congruent to each other, and each part is similar to the original figure, then the original figure is called a **rep-tile**. For instance, every square is a rep-tile, since it can be cut into four congruent squares as shown at right, and each of these squares is, of course, similar to the original square.

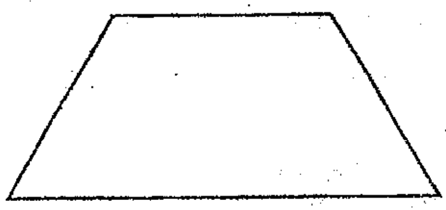


Dissect each polygon into four congruent polygons, each of which is similar to the original.

1.



2.



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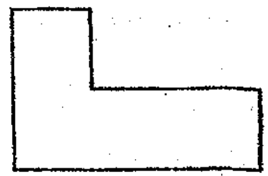

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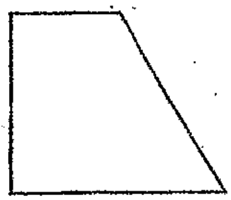
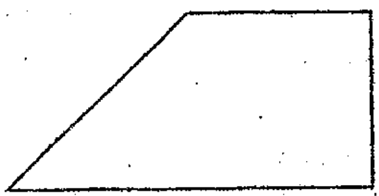
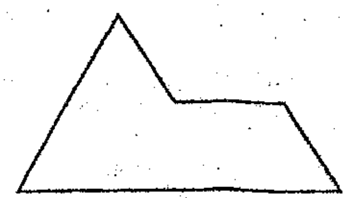


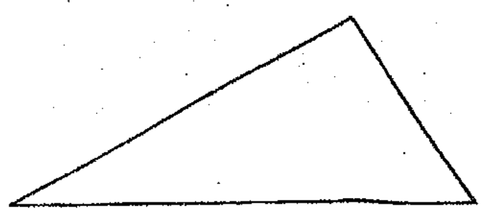
5.



6.



7. The figures in Exercises 1–6 are called *rep-4 figures* because the dissection results in four similar figures. Every triangle, no matter what its shape, is a rep-4 figure. Use the scalene triangle at right to demonstrate how any triangle can be dissected into four congruent triangles that are similar to it.



8. There is exactly one type of triangle that is a rep-2 figure. What is it? Draw the triangle and its rep-2 dissection in the space at right.

9. Besides the triangle in Exercise 8, the only other known rep-2 polygon is a parallelogram in which the lengths of the sides have a specific relationship. What is that relationship? (Hint: Let a and b represent the lengths of two adjacent sides of the parallelogram. Set up and solve a proportion.)

____

10. Give the dimensions of a rep-n rectangle, where n is any integer greater than 1.

____

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Enrichment
8.3 Similarity and Special Triangles

In your study of geometry, you have learned about several special types of triangles, most notably equilateral triangles, isosceles triangles, and right triangles. On this page, you will investigate conditions under which two special triangles are similar to each other.

1. Justify the following statement: All equilateral triangles are similar.

____

____

Sketch a counterexample to show why each of these statements is false.

2. All isosceles triangles are similar.

3. All right triangles are similar.

Tell whether each statement is *True* or *False*. If the statement is false, sketch a counterexample. If the statement is true, write a justification to support it. Show your work on a separate sheet of paper.

4. Two isosceles triangles are similar if a base angle of one is congruent to a base angle of the other. ____________

5. Two isosceles triangles are similar if the vertex angle of one is congruent to the vertex angle of the other. ____________

6. Two isosceles triangles are similar if the legs of one are proportional to the legs of the other. ____________

7. Two isosceles triangles are similar if the base and a leg of one are proportional to the base and a leg of the other. ____________

Find three different ways to fill in the blank to make a true statement:

Two right triangles are similar if ____________.

8. ____

9. ____

10. ____

11. On a separate sheet of paper, write a justification of each statement in Exercises 8–10.

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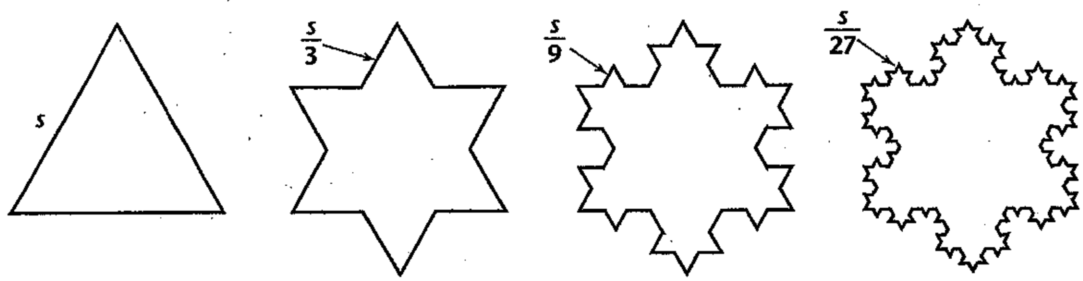
Enrichment
11.6 *Finding the Area of the Koch Snowflake*

The diagram below shows the first four stages in the construction of the *Koch Snowflake*. Perhaps you have already worked with this figure. If so, you may have developed some intuition about its area. On this page you will see how you can use algebraic techniques to actually calculate the area.

> *Reminder:*
>
> The area of an equilateral triangle with side length s is $\frac{\sqrt{3}}{4}s^2$.



stage 0 | stage 1 | stage 2 | stage 3

1. What is the area, A_0, of the snowflake at stage 0? $A_0 =$ ____________

2. a. What is the area of each triangle *added* at stage 1? ____________

   b. How many triangles are added at stage 1? ____________

   c. What is the total area, A_1, of the snowflake at stage 1? $A_1 =$ ______ + ______

3. What is the total area, A_2, of the snowflake at stage 2? $A_2 =$ ______ + ______ + ______

4. What is the area, A_3, of the snowflake at stage 3? $A_3 =$ ____ + ____ + ____ + ____

5. After several calculations, it can be shown that a formula for the area of the snowflake at any stage n, where n is greater than 0, is $A_n = \frac{\sqrt{3}}{4}s^2 + \frac{\sqrt{3}}{12}\left(1 + \frac{4}{9} + \frac{4^2}{9^2} + \cdots + \frac{4^{n-1}}{9^{n-1}} + \cdots\right)s^2$. On a separate sheet of paper, verify that your answers to Exercise 2–4 satisfy this formula.

6. In Exercise 5, $\left(1 + \frac{4}{9} + \frac{4^2}{9^2} + \cdots + \frac{4^{n-1}}{9^{n-1}} + \cdots\right)$ is an *infinite geometric series*. The ratio between every set of consecutive terms is $\frac{4}{9}$. A formula for finding a sum like this is $S = \frac{a}{1-r}$, where a is the first term and r is the ratio between terms. Use this formula to find the sum. ____________

7. Substitute your answer to Exercise 6 into the formula in Exercise 5. Simplify the result. What is the area of the Koch Snowflake? ____________

8. Use the method shown above to verify that the area of the Sierpinski Gasket is 0 when the initial shape is an equilateral triangle. Show your work on a separate sheet of paper.