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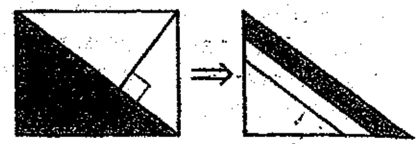
Enrichment
8.5 *Similar Triangles Within a Right Triangle*

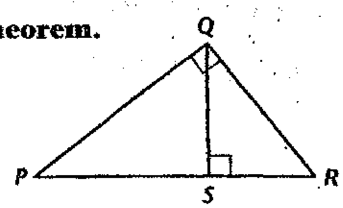
Take a rectangular sheet of paper and draw the segments shown at right. Cut out the three triangles and align them as shown. If your work is accurate, the triangles should appear similar. In fact, you have demonstrated the following important theorem about right triangles.



RIGHT TRIANGLE ALTITUDE THEOREM

In a right triangle, the altitude from the vertex of the right angle to the hypotenuse forms two triangles that are similar to the given triangle and to each other.

Complete the following proof of the Right Triangle Altitude Theorem.



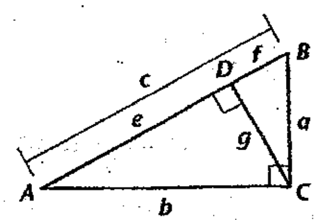
Given: $\angle PQR$ is a right angle.
$\overline{QS}$ is an altitude of $\triangle PQR$.

Prove: $\triangle PSQ \sim \triangle PQR$; $\triangle QSR \sim \triangle PQR$; $\triangle PSQ \sim \triangle QSR$

STATEMENTS	REASONS
$\angle PQR$ is a right angle. $\overline{QS}$ is an altitude of $\triangle PQR$.	1. ____________
$\overline{QS} \perp \overline{PR}$	2. ____________
$\angle PSQ$ and $\angle QSR$ are right angles.	3. ____________
$m\angle PSQ = m\angle QSR = m\angle PQR = 90°$	4. ____________
$\angle PSQ \cong \angle PQR$; $\angle QSR \cong \angle PQR$	5. ____________
$\angle P \cong \angle P$; $\angle R \cong \angle R$	6. ____________
$\triangle PSQ \sim \triangle PQR$; $\triangle QSR \sim \triangle PQR$	7. ____________
$\triangle PSQ \sim \triangle QSR$	8. ____________

9. **a.** Name all pairs of similar triangles in the figure at right.

____



b. Complete each proportion: $\frac{a}{f} = \frac{c}{\square}$ and $\frac{b}{e} = \frac{c}{\square}$

c. On a separate sheet of paper, write a proof of the Pythagorean Theorem that utilizes the figure above and your results from parts **a** and **b**. (Hint: Apply the Cross-Multiplication Property to the proportions and add the resulting equations.)

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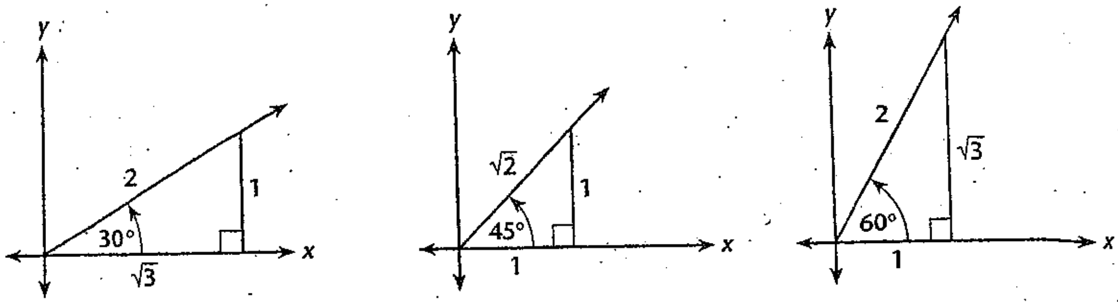
10.3 Exact Trigonometric Ratios for Special Angles

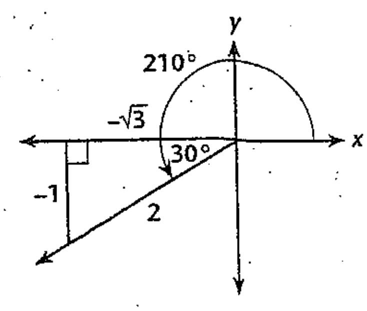
The figures below show two 30-60-90 triangles and a 45-45-90 triangle positioned in the first quadrant of a coordinate plane, with convenient measures chosen for the lengths of the sides. You can use these figures to calculate exact, rather than approximated, trigonometric ratios for angles of 30°, 45°, and 60°.



Refer to the figures above. Complete the table by giving *exact values* for each trigonometric ratio. (Hint: Study the values that have already been entered into the table.)

	θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
1.	30°		$\frac{\sqrt{3}}{2}$	
2.	45°	$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$		
3.	60°		$\frac{1}{2}$	

To find exact values of the trigonometric ratios for special angles in other quadrants, you can use the special right triangles as *reference triangles*. You simply need to keep in mind whether the ratio is positive or negative in the given quadrant. For example, the figure at right shows a 30-60-90 triangle positioned as the reference triangle for a 210° angle, which is in the third quadrant.



Refer to the figure at the right. Give the *exact value* of each ratio.

4. sin 210° ____________ **5.** cos 210° ____________ **6.** tan 210° ____________

Using the special right triangles as reference triangles, give the exact value of each ratio.

7. cos 150° ____________ **8.** tan 300° ____________ **9.** sin 225° ____________

10. sin (−60°) ____________ **11.** tan (−135°) ____________ **12.** cos (−300°) ____________

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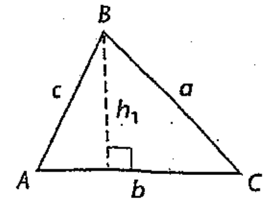
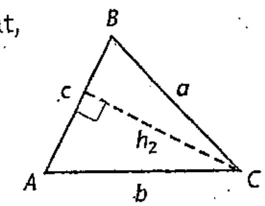
Enrichment
10.4 Alternative Area Formulas for Triangles

In the figure at right, the area of $\triangle ABC$ can be found by using the formula Area $= \frac{1}{2}bh_1$. You know that $\sin A = \frac{h_1}{c}$. Solving this equation for h_1, you obtain $h_1 = c \sin A$. Substituting the expression $c \sin A$ for h_1 into the area formula, you obtain an alternative formula for the area of $\triangle ABC$:

Area $= \frac{1}{2}bc \sin A$.

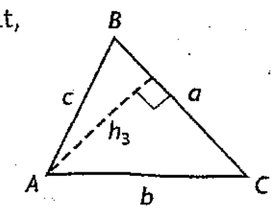


1. Using the figure at right, write an alternative formula for the area of $\triangle ABC$ that involves $\sin B$.



Area = ______________________

2. Using the figure at right, write an alternative formula for the area of $\triangle ABC$ that involves $\sin C$.



Area = ______________________

Use the above formulas to find the area of $\triangle ABC$ that has the given measures. Round your answers to the nearest tenth.

3. $a = 5$ inches, $b = 8$ inches, $m\angle C = 47°$

Area = ______________________

4. $b = 1.05$ meters, $c = 4.2$ meters, $m\angle A = 122°$

Area = ______________________

The above formulas are useful when you know the lengths of two sides of a triangle and the measure of the included angle. You can also derive alternative formulas for the area of a triangle when you know the measures of two angles and the length of one side.

5. a. According to the Law of Sines, given triangle ABC, $\frac{\sin A}{a} = \frac{\sin B}{b}$. Solve this equation for b.

$b =$ ______________________

b. Substitute the expression for b from part **a** into the formula Area $= \frac{1}{2}ab \sin C$. Simplify.

Area = ______________________

6. Derive a formula for area similar to that in Exercise 5 by starting with $\frac{\sin B}{b} = \frac{\sin C}{c}$.

Area = ______________________

7. Derive a formula for area similar to that in Exercise 5 by starting with $\frac{\sin A}{a} = \frac{\sin C}{c}$.

Area = ______________________

Use your formulas from Exercises 5–7 to find the area of triangle ABC that has the given measures. Round your answers to the nearest tenth.

8. $a = 6$ feet, $m\angle B = 72°$, $m\angle C = 29°$

Area = ______________________

9. $c = 4.8$ centimeters, $m\angle A = 143°$, $m\angle C = 15°$

Area = ______________________

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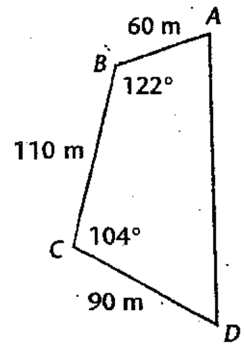
10.5 The Law of Sines, the Law of Cosines, and Diagonals

As you know, every polygonal region can be divided into triangular regions by drawing one or more diagonals. When the Law of Sines and the Law of Cosines can be applied to these triangular regions, you can determine many more facts about the polygon than might have seemed possible at first glance.



For example, the figure at right shows a plot of land. A surveyor has been able to find the labeled measures by using conventional surveying instruments. The fourth side of the plot is inaccessible.

For Exercises 1–6, refer to the figure at right.

1. Draw $\overline{AC}$. Use the Law of Cosines to find its length. Round to the nearest tenth of a meter. $AC =$ ____________

2. Use the Law of Sines to find the measure of $\angle BCA$. Round to the nearest degree. $m\angle BCA =$ ____________

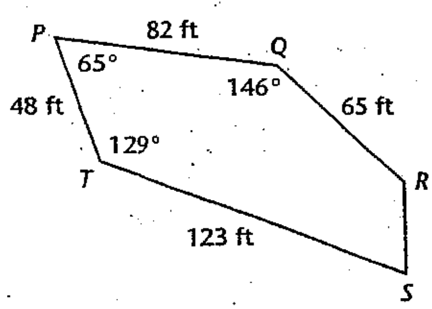
3. What is the measure of $\angle ACD$? $m\angle ACD \approx$ ____________

4. Use the Law of Cosines to find the length of $\overline{AD}$. Round to the nearest tenth of a meter. $AD \approx$ ____________

5. Use the Law of Sines to find the measure of $\angle CDA$. Round to the nearest whole number of degrees. $m\angle CDA \approx$ ____________

6. What is the measure of $\angle BAD$? $m\angle BAD \approx$ ____________

Refer to the figure at right. Find each measure.



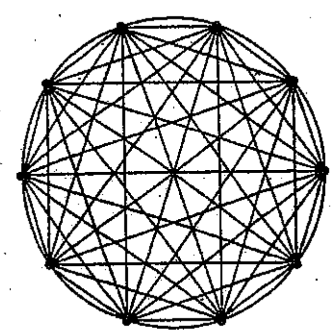
7. $m\angle R$ ____________

8. $m\angle S$ ____________

9. RS ____________

10. the area of $PQRST$ ____________

11. A circle has been drawn on a block of wood, and ten pegs have been pounded into the wood so that they are equally spaced around the circle. A student is planning to make a "string art" design by looping brightly colored yarn between the pegs in the manner shown at right. The distance between each pair of consecutive pegs is 3 inches. Use the Law of Sines and/or the Law of Cosines to find the total amount of yarn that the student will need for the design.



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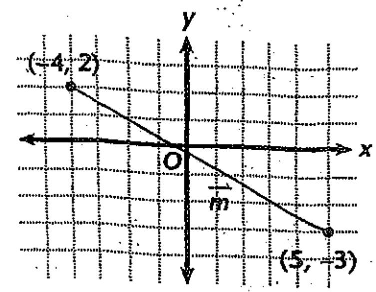
Enrichment

10.6 Vectors on a Coordinate Plane

When a vector is on a coordinate plane, it can be represented by an ordered pair. If its tail, or **initial point**, has coordinates (x_1, y_1) and its head, or **terminal point**, has coordinates (x_2, y_2), then its ordered pair is $(x_2 - x_1, y_2 - y_1)$. For example, the ordered pair for vector $\overrightarrow{m}$ on the coordinate plane at right is $(-4 - 5, 2 - [-3])$, or $(-9, 5)$.



Find the ordered pair representation of a vector with initial point *J* and terminal point *K*.

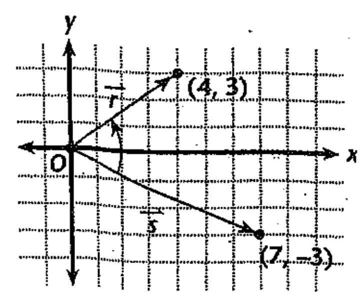
1. $J(5, 2), K(9, 3)$ ____________ 2. $J(6, -8), K(2, -1)$ ____________

The **dot product** of two vectors $\overrightarrow{u} = (a_1, b_1)$ and $\overrightarrow{v} = (a_2, b_2)$ is denoted $\overrightarrow{u} \cdot \overrightarrow{v}$ and is defined by the rule $\overrightarrow{u} \cdot \overrightarrow{v} = a_1a_2 + b_1b_2$. For example, if $\overrightarrow{u} = (-3, 7)$ and $\overrightarrow{v} = (-2, -1)$, then the dot product $\overrightarrow{u} \cdot \overrightarrow{v}$ is $(-3)(-2) + 7(-1) = -1$. Notice that the dot product is a real number. It is *not* another vector.

Let $\overrightarrow{u} = (1, 3)$, $\overrightarrow{v} = (-2, 4)$, and $\overrightarrow{w} = (-8, -4)$. Find each dot product.

3. $\overrightarrow{u} \cdot \overrightarrow{v}$ ____________ 4. $\overrightarrow{u} \cdot \overrightarrow{w}$ ____________ 5. $\overrightarrow{v} \cdot \overrightarrow{w}$ ____________

The dot product provides a method for calculating the angle between two vectors. For convenience, consider two vectors, $\overrightarrow{r}$ and $\overrightarrow{s}$, in *standard position,* that is, with their initial points at the origin. Let θ be the angle between $\overrightarrow{r}$ and $\overrightarrow{s}$. Then the Law of Cosines can be used to prove that $\cos \theta = \dfrac{\overrightarrow{r} \cdot \overrightarrow{s}}{|\overrightarrow{r}||\overrightarrow{s}|}$. So $\theta = \cos^{-1} \dfrac{\overrightarrow{r} \cdot \overrightarrow{s}}{|\overrightarrow{r}||\overrightarrow{s}|}$.



Refering to the figure at right, represent each expression below numerically.

6. $\overrightarrow{r}$ ____________ 7. $\overrightarrow{s}$ ____________ 8. $\overrightarrow{r} \cdot \overrightarrow{s}$ ____________

9. $|\overrightarrow{r}|$ ____________ 10. $|\overrightarrow{s}|$ ____________ 11. $|\overrightarrow{r}||\overrightarrow{s}|$ ____________

12. $\cos \theta$ ____________ 13. θ ____________

Find the angle between each pair of vectors.

14. $\overrightarrow{c} = (-3, 1)$, $\overrightarrow{d} = (-6, -3)$ ____________ 15. $\overrightarrow{p} = (-1, -2)$, $\overrightarrow{q} = (4, 2)$ ____________

16. The dot product of vectors $\overrightarrow{j}$ and $\overrightarrow{k}$ is 0. What is the relationship between $\overrightarrow{j}$ and $\overrightarrow{k}$? Explain.

____

____