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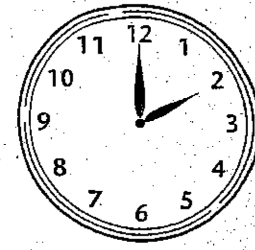
Enrichment

5.3 Circumference, Rotations, and Functions

As the minute hand of a clock moves, its tip traces a circular path. The radius of the path is the length of the minute hand. How far does the tip travel in one hour? Clearly, the distance depends on the length of the hand. For example:

- If the length is 4 inches, then the distance traveled is $2\pi(4)$ inches, or about 25.1 inches.
- If the length is 3 feet, then the distance traveled is $2\pi(3)$ feet, or about 18.8 feet.



Apply the formula for the circumference of a circle with radius r: $C = 2\pi r$.

The distance the tip travels is a *function* of the length of the minute hand. If this function is named *f*, a rule for the function is $f(x) = 2\pi x$, where *x* is the length of the minute hand.

Write a rule for each specified function.

1. The function *g* relates the distance that the tip of the minute hand travels in one minute to the length of the minute hand, *x*. ____________

2. The function *h* relates the distance that the tip of the hour hand travels in one hour to the length of the hour hand, *y*. ____________

3. The function *j* relates the distance that the tip of a 3-inch hour hand travels to the number of hours, *z*. ____________

4. The function *k* relates the distance that the tip of a 5-inch minute hand travels to the number of days, *v*. ____________

In writing rules for the above functions, you used common knowledge about rotations on a clock: The minute hand makes one rotation every hour, and the hour hand makes one rotation every 12 hours. In other situations, you must be given information about rotations before you can write a function rule.

5. A Ferris wheel with a radius of 4 yards is rotating at a constant speed. It takes 90 seconds to make one complete rotation. Write a rule for the function, *r*, that relates the number of feet that a rider travels to the number of minutes, *t*, that the wheel has been rotating. ____________

6. The diameter of each wheel of a bicycle is 26 inches, and the wheel rotates 5 times every 2 seconds. Write a rule for the function, *m*, that relates the number of minutes that the bicycle travels to the number of miles traveled, *d*. ____________

7. Describe an original rotation situation that can be specified by the function rule $f(x) = 18\pi x$.

____

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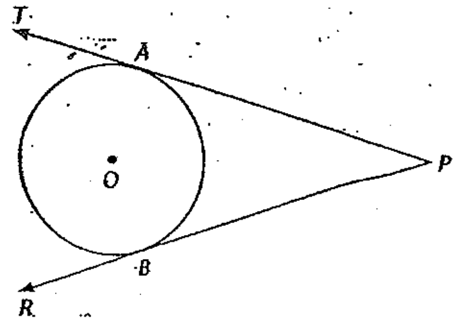
Enrichment

9.2 Tangents to Circles

Write a two-column proof using the plan shown.



Given: Circle O with $\overrightarrow{PT}$ tangent at A and $\overrightarrow{PR}$ tangent at B

Prove: $\overline{OP}$ bisects $\angle P$.

Plan: Draw radii $\overline{OA}$ and $\overline{OB}$, and prove $\triangle OAP \cong \triangle OBP$.

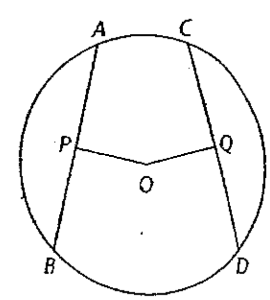
Proof:

Statements	Reasons
Circle O with $\overrightarrow{PT}$ tangent at A and $\overrightarrow{PR}$ tangent at B	1. ______
Draw radii $\overline{OA}$ and $\overline{OB}$.	2. ______
$\overline{OA} \perp \overline{PA}$, $\overline{OB} \perp \overline{PB}$	3. ______
$\angle OAP$ and $\angle OBP$ are right $\angle$s.	4. ______
$\triangle OAP$ and $\triangle OBP$ are right $\triangle$s.	5. ______
$\overline{OP} \cong$ ______	6. ______
$\overline{OA} \cong$ ______	7. ______
$\triangle OAP \cong \triangle OBP$	8. ______
$\angle APO \cong \angle BPO$	9. ______
$\overline{OP}$ bisects $\angle P$.	10. ______

11. Write the theorem you proved in Exercises 1–10.

____

12. Write a paragraph proof on your own paper using the plan shown.



Given: In circle O, $\overline{AB}$ and $\overline{CD}$ are chords; $\overline{OP} \perp \overline{AB}$, $\overline{OQ} \perp \overline{CD}$; $\overline{OP} \cong \overline{OQ}$

Prove: $\overline{AB} \cong \overline{CD}$

Plan: Draw radii $\overline{OB}$ and $\overline{OD}$, and show $\triangle OPB \cong \triangle OQD$ to get $\overline{BP} \cong \overline{DQ}$.

13. Write the theorem you proved in Exercise 12.

____

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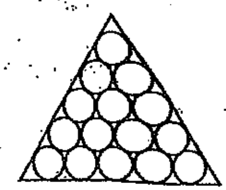
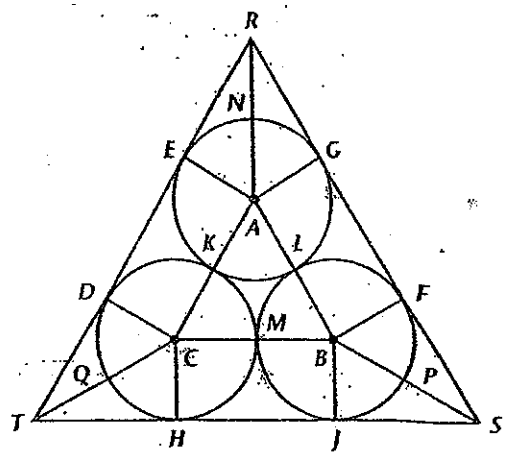
Enrichment
9.4 Racking Billiard Balls

A regulation pocket billiard ball is a perfect sphere with a diameter of 2.25 inches, and a tolerance of 0.005 inch. At the start of a game of pocket billiards, the fifteen balls must be arranged in five rows in a triangular rack as shown at right. On this page, you will see how the properties of circles determine the shape and size of the rack.



Below at right is a figure depicting just two rows of billiard balls in a rack. On a separate sheet of paper, justify each statement about this figure.



1. $\triangle ABC$ is an equilateral triangle.
2. $ACDE$ is a rectangle.
3. $m\widehat{KL} = 60°$
4. $m\overline{EK} = m\overline{GL} = 90°$
5. $m\widehat{ENG} = 120°$
6. $m\angle ERG = 60°$
7. $\triangle AER \cong \triangle AGR$
8. $m\angle ERA = 30°$
9. $AC = 2.25$ inches
10. $ED = 2.25$ in.
11. $AE = 1.125$ inches
12. $ER = (1.125)\sqrt{3}$ inches
13. $TD = (1.125)\sqrt{3}$ inches
14. $TR = (2.25 + 2.25\sqrt{3})$ inches ≈ 6.1 inches

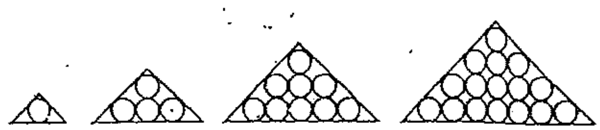
Following similar reasoning, $m\angle HTD = m\angle FSJ = 60°$ and $RS = ST \approx 6.1$ inches. So a triangular rack for two rows of pocket billiard balls would be an equilateral triangle with sides that are each slightly longer than 6.1 inches in length.

Suppose that a rack shaped like an equilateral triangle encloses the given number of pocket billiard balls. Find the length of each side of the rack. (Hint: How many rows of balls will there be?)

15. 6 balls ____________
16. 10 balls ____________
17. 15 balls ____________

18. Write an expression for the length, in inches, of each side of the equilateral triangular rack that would enclose n rows of pocket billiard balls in the manner shown above. ____________

19. Suppose that billiard balls of diameter d inches were racked in the pattern shown at right. Describe the rack that would enclose n rows of billiard balls in this way.



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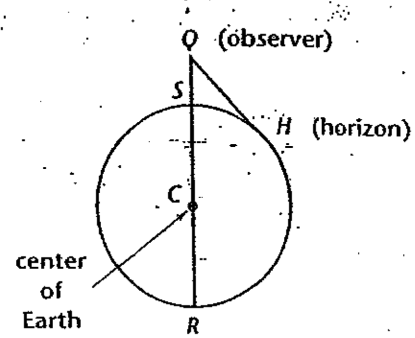
Enrichment
9.5 Finding the Distance to the Horizon

For an observer at a point O above Earth, the horizon is the place where Earth appears to "meet the sky." The higher above Earth's surface the observer is, the farther away the horizon appears to be. It may surprise you to learn that you can calculate this distance to the horizon by applying your knowledge of tangents and secants.



Refer to the diagram of Earth at right.

1. Name the segment that represents each measure.

   a. the diameter of Earth ____________

   b. the observer's altitude above Earth's surface ____________

   c. the distance the observer can see to the horizon ____________

2. Justify the following equation: $(OH)^2 = OR \cdot OS$

____

____

When the observer's altitude above Earth's surface is small relative to the diameter of Earth, you can replace OR with RS in the equation from Exercise 2. Then, since the diameter of Earth is approximately 7920 miles, you obtain the formula for OH shown at right. In this formula, the unit for both OH and OS is miles.

$$(OH)^2 = OR \cdot OS$$
$$(OH)^2 \approx RS \cdot OS$$
$$(OH)^2 \approx 7920 \cdot OS$$
$$OH \approx \sqrt{7920 \cdot OS}$$

Use the formula above to find the distance to the horizon for each altitude. Assume that it is a clear day and that the view is not obstructed. Round answers to the nearest tenth of a mile.

3. 2.5 miles ____________

4. 30,000 feet ____________

5. Rewrite the formula above so that you can input OS as a number of feet and find the distance to the horizon in *miles*.

   $\left(\text{Hint: } OH \text{ miles} = \sqrt{7920 \text{ miles} \cdot \frac{1 \text{ mile}}{? \text{ feet}} \cdot OS \text{ feet}}\right)$ ____________

Use your formula from Exercise 5 to find the distance to the horizon for each altitude. Assume that it is a clear day and that the view is not obstructed. Round answers to the nearest tenth of a mile.

6. 10 feet ____________

7. 200 feet ____________

Find the altitude above Earth's surface that an observer must attain in order to see the given distance to the horizon. Round answers to the nearest tenth.

8. 1 mile ____________

9. 300 miles ____________

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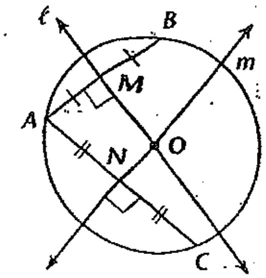
Enrichment

9.6 Finding the Equation of a Circle Given Three Points



Any three noncollinear points determine a circle. You have learned that you can locate the center of that circle by drawing two chords and constructing the perpendicular bisector of each. As a result of Theorem 9.2.5, the intersection of these perpendicular bisectors will be the center of the circle. Then the distance between the center and any one of the given points gives you the radius of the circle.

For Exercises 1–5, refer to the points $A(-4, -3)$, $B(1, 2)$, and $C(5, -6)$. Use the construction shown at right to help you visualize this situation.

1. **a.** Find the coordinates of M, the midpoint of $\overline{AB}$. ____________

b. What is the slope of $\overline{AB}$? ____________

c. What is the slope of any line that is perpendicular to $\overline{AB}$? ____________

d. Write an equation in slope-intercept form for line ℓ, the perpendicular bisector of $\overline{AB}$. ____________

2. Write an equation in slope-intercept form for line m, the perpendicular bisector of $\overline{AC}$. ____________

3. **a.** Use your equations from Exercises 1 and 2 to write a system of linear equations. { ____________

b. Solve the system. Identify the solution as point O. ____________

4. What is the distance between point O and:

a. point A? ____________ **b.** point B? ____________ **c.** point C? ____________

5. Write an equation in standard form for the circle with center at point O that passes through points A, B, and C. ____________

Write an equation in standard form for the circle that passes through the given points.

6. $P(-10, -1)$, $Q(4, -3)$, and $R(8, 5)$ ____________

7. $J(-6, -1)$, $K(0, 5)$, and $L(-6, 7)$ ____________

8. $F(-9, -4)$, $G(0, -1)$, and $H(3, -10)$ ____________

9. Write equations in standard form for *all* the circles with radius 5 that pass through the points $D(6, 4)$ and $E(2, -4)$. (Hint: Graph points D and E and draw the perpendicular bisector of $\overline{DE}$.)

____

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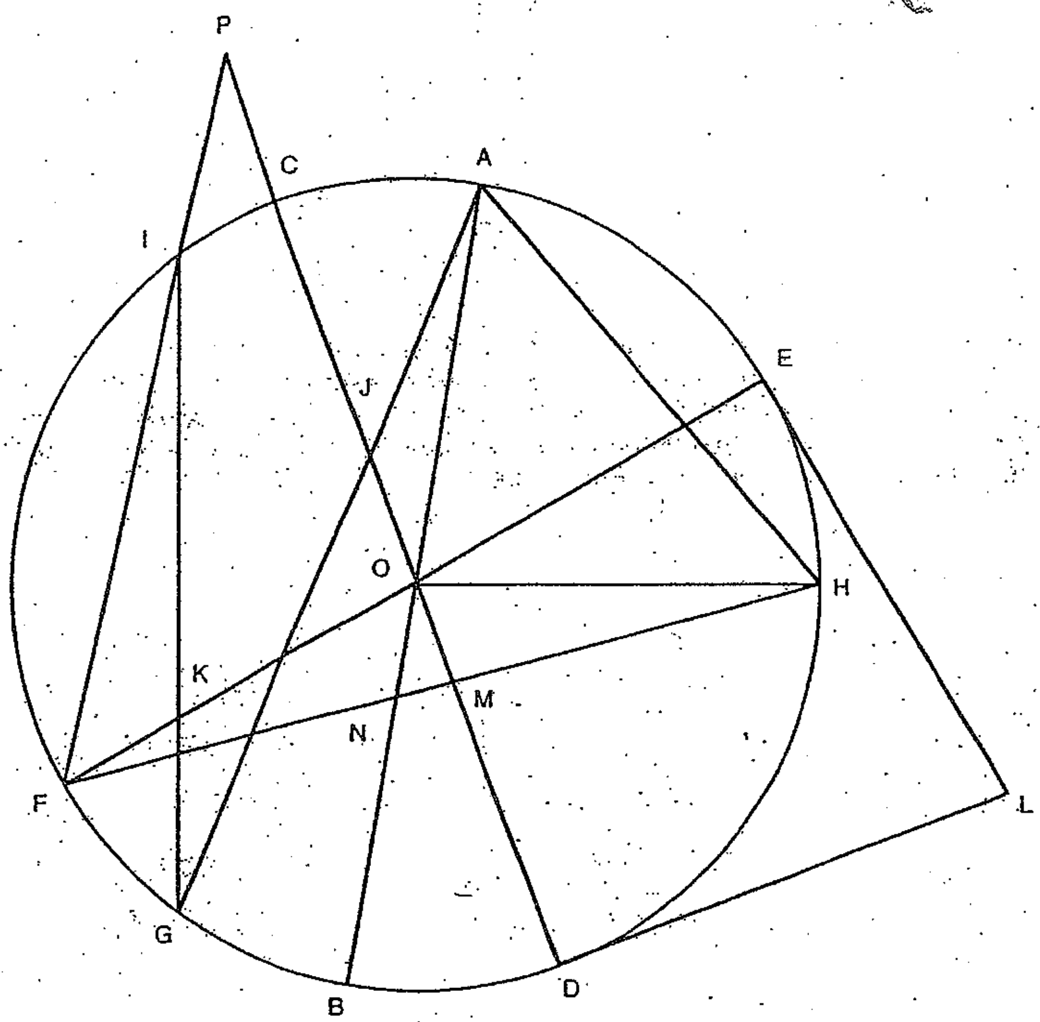
4–10B The Biggest Circle Puzzle

Using the information you were given and your knowledge of circles, fill in the measures of the angles, arcs, and segments on the diagram below.



$\overline{LD}$ and $\overline{LE}$ are tangent to the circle.

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4–10A The Biggest Circle Puzzle

You are given the following information:

O is the center of the circle.

$\overline{AB}$, $\overline{CD}$, and $\overline{EF}$ are diameters.

$m\overset{\frown}{GB} = 26$	$m\angle AOC = 30$
$m\angle OAH = 50$	$m\angle AHF = 65$
$m\angle IGA = 24$	$IF = 10.5$
$PI = 4.5$	$PC = 3.4$
$CM = 10$	$HM = 7.5$

Label the circle on the accompanying worksheet with the given information, then find the measure of each angle, arc, and segment below. Write the measure of each angle, arc, and segment as you find it on the diagram and use tick marks to designate congruent angles. Check your work as you proceed. Do not make any assumptions. Record your answers below. (Round to the nearest whole number where necessary.)

1. $m\overset{\frown}{CB} =$ ______
2. $m\angle AJC =$ ______
3. $m\overset{\frown}{CG} =$ ______
4. $m\angle FOB =$ ______
5. $m\overset{\frown}{FG} =$ ______
6. $m\angle FPC =$ ______
7. $m\angle FIG =$ ______
8. $m\overset{\frown}{FI} =$ ______
9. $m\angle IKF =$ ______
10. $m\overset{\frown}{AE} =$ ______
11. $m\angle ELD =$ ______
12. $m\overline{CD} =$ ______
13. $m\angle COF =$ ______
14. $m\overline{FM} =$ ______
15. $m\angle FNB =$ ______