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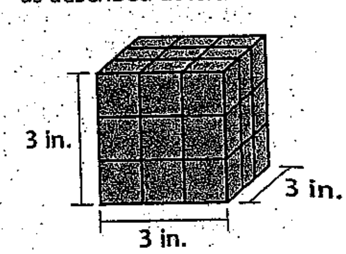
Enrichment
2.1 Three-Dimensional Proofs

Just as with figures in a plane, you can write proofs about three-dimensional figures. For example, consider the following problem.

A cube made of solid wood has edges of length n inches, where n is a whole number greater than 1. The cube is painted red, then cut into small *unit cubes* with edges of length 1 inch. The result is that some of the unit cubes have exactly three red faces, some have exactly two red faces, some have exactly one red face, and the rest have no red faces at all.

This cube with edges of length 3 inches has been cut as described at left.



Complete this table for cubes that are cut as described above.

	Length of one edge in inches	Total number of unit cubes	Number of unit cubes with exactly 3 red faces	Number of unit cubes with exactly 2 red faces	Number of unit cubes with exactly 1 red face	Number of unit cubes with no red faces
1.	2					
2.	3					
3.	4					
4.	5					
5.	6					

Now consider a solid wooden cube with edge of length n inches, as described above. Write an algebraic expression that represents the number of each type of unit cube that results. On a separate sheet of paper, write a convincing argument to support each answer.

6. the total number ________

7. exactly 3 red faces ________

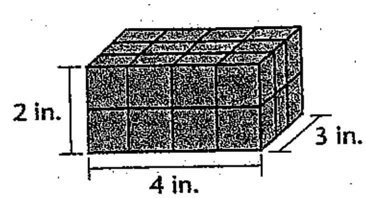
8. exactly 2 red faces ________

9. exactly 1 red face ________

10. no red faces ________

11. A box made of solid wood has edges of length n inches, $(n + 1)$ inches, and $(n + 2)$ inches, where n is a whole number greater than 1. The box is painted red, then cut into several unit cubes with edges of length 1 inch. Using your own paper, conduct an investigation to find how many of the unit cubes have exactly three red faces, exactly two red faces, exactly one red face, and no red faces.

This box with edges of length 2 inches, 3 inches, and 4 inches has been cut as described at left.



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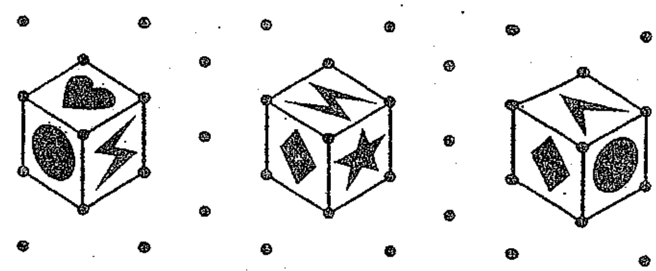
Enrichment

6.1 Isometric Drawings and Nets

A **net** is a two-dimensional figure that, when folded, forms the surface of a three-dimensional figure. Often a set of isometric drawings can give you sufficient information to draw a net for a geometric figure. For instance, consider the drawings at right, which show three views of the same cube.



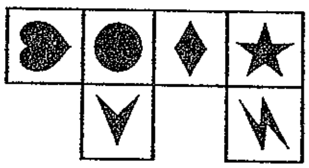
Each of the following is a face of the cube shown above. In the space to its right, make a sketch of the face of the cube that is directly opposite the given face.

1. 

2. 

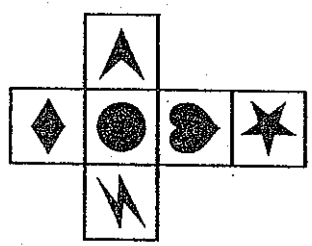
3. 

Explain why each of the following is *not* a net for the cube shown above.

4. 

____

____

5. 

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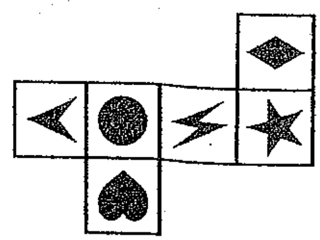
6. 

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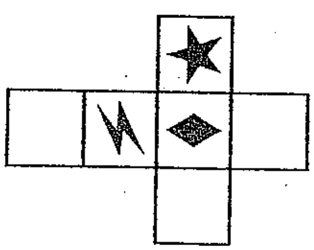
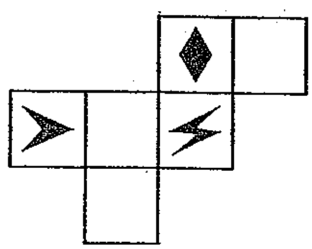
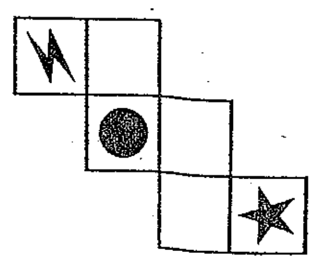
____

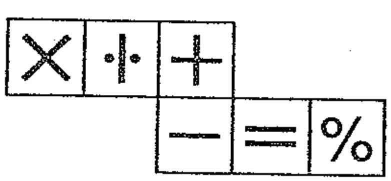
Complete each figure so that it *is* a net for the cube shown above.

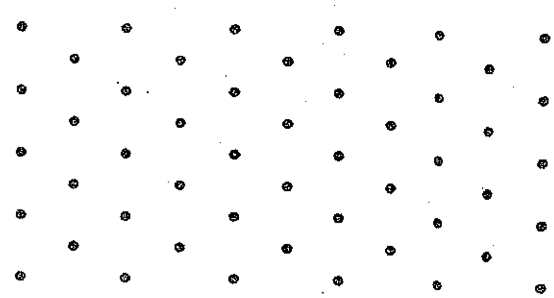
7. 

8. 

9. 

10. Below is a net for a cube. In the space at right, make a set of isometric drawings that describe the cube completely.





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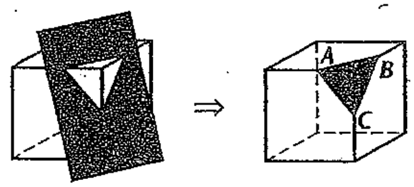
Enrichment

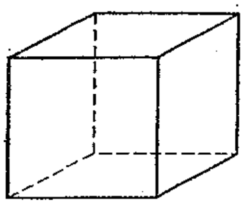
6.2 Planes and Cross Sections

he intersection of a plane and a three-dimensional figure called a **cross section** of the three-dimensional figure. or instance, the figure at right shows plane *P* intersecting cube. The resulting cross section is a triangle, which is lentified in the figure at the far right as $\triangle ABC$.



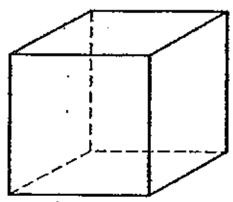
: it possible for the intersection of a plane and a cube to have ıe given shape? If it is possible, sketch such a cross section on ıe cube. If it is not possible, write *not possible* on the line below.

square



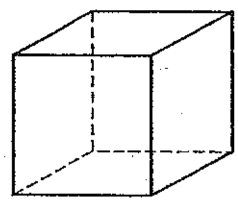
____

2. non-square rectangle



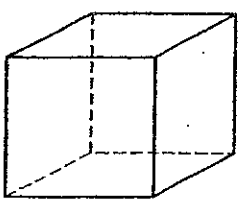
____

3. non-square rhombus



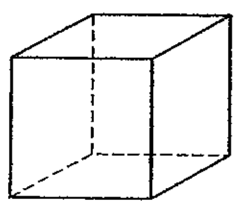
____

trapezoid



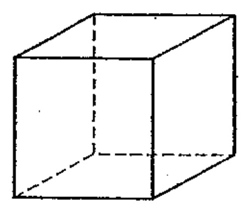
____

5. pentagon



____

6. hexagon



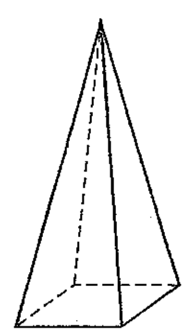
____

Tell whether the following statement is always, sometimes, or never true: *If two parallel planes intersect a cube, the resulting cross sections are congruent.* Justify your answer.

____

____

The figure at right is a *pyramid*. It rests on a square face that is called its *base*. The other four faces are congruent isosceles triangles. Name all possible shapes for a cross section of this figure formed by an intersecting plane that is:



a. parallel to the base.

____

____

b. perpendicular to the base.

____

____

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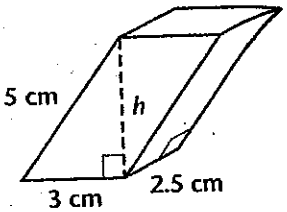
7.2 Surface Area of Oblique Prisms

As you have learned, there is no simple general formula for the surface area of an oblique prism. However, you are often given enough additional information about a figure to make it possible to calculate the surface area of a particular prism.



Exercises 1–4 refer to the oblique rectangular prism at right.

1. a. What type of figure bounds each base of the prism? __________

   b. What is the area of each base? __________

2. a. What type of figure bounds the "left" and "right" faces of the prism? __________

   b. What is the area of each of these faces? __________

3. a. What type of figure bounds the "front" and "back" faces of the prism? __________

   b. What is the value of h? __________

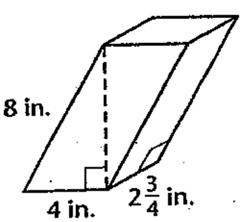
   c. What is the area of each of these faces? __________

4. a. What is the total surface area of the prism? __________

   b. What is the volume of the prism? __________

Find the surface area, S, and volume, V, of each oblique rectangular prism. Give answers in exact form.

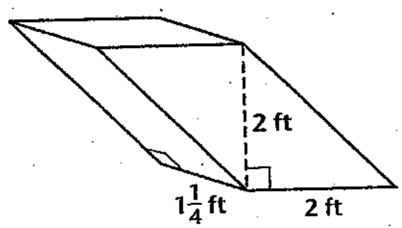
5.



$S =$ __________

$V =$ __________

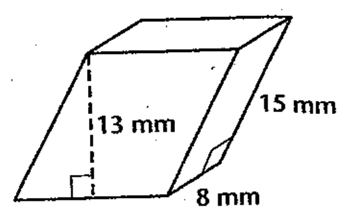
6.



$S =$ __________

$V =$ __________

7.



$S =$ __________

$V =$ __________

8. Each lateral face of an oblique square prism is bounded by a rhombus in which the measure of one angle is 60°. Write formulas for the surface area, S, and volume, V, of this prism in terms of the length of one side of a square base, n. (Hint: Draw a net and assemble it to make a model.)

$S =$ __________ $V =$ __________

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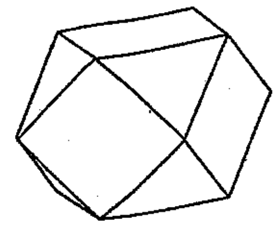
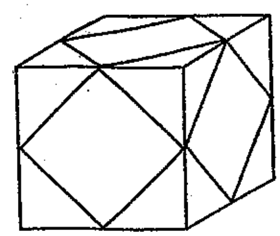
7.3 *Surface Area and Volume of Semiregular Polyhedra*

A **semiregular polyhedron** is a convex polyhedron whose faces are bounded by two or more types of regular polygons in such a way that the arrangement of polygons at each vertex of the polyhedron is identical.



The figure at right is a semiregular polyhedron called a *cuboctahedron*. Its faces are bounded by equilateral triangles and squares. You can think of it as the figure obtained if you "cut off" eight congruent pieces from a cube in the manner shown.



1. **a.** How many square faces does a cuboctahedron have? ____________

b. How many triangular faces does a cuboctahedron have? ____________

2. What type of figure is each piece that is "cut off" from the original cube? ____________

Suppose that the length of each edge of a cuboctahedron is 10 inches.

3. **a.** What is the area of each square face? ____________

b. What is the area of each triangular face? ____________

c. What is the total surface area? ____________

4. **a.** What is the length of each edge of the original cube from which the cuboctahedron was "cut?" ____________

b. What is the volume of this cube? ____________

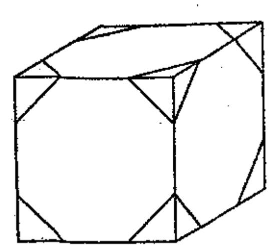
c. What is the volume of each piece that was "cut off" from the cube? ____________

d. What is the volume of the cuboctahedron? ____________

5. Generalize your results from Exercises 3 and 4 to write formulas for the surface area, S, and volume, V, of a cuboctahedron with edge of length n.

$S =$ ____________ $V =$ ____________

6. When eight congruent pieces are cut from a cube in the manner shown at right, the result is a semiregular polyhedron called a *truncated cube*. Write formulas for the surface area, S, and volume, V, of a truncated cube with edge of length m.



$S =$ ____________

$V =$ ____________

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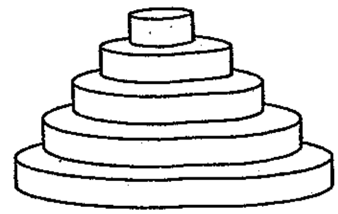
Enrichment
7.4 Surface Area and Volume of a "Wedding Cake"

The figure at right is a stack of right cylinders. The height of each cylinder in the stack is one unit. The radius of the cylinder at the top is one unit, and the radius of each other cylinder is one unit greater than the radius of the cylinder directly above it. An arrangement of cylinders like this is sometimes called a "wedding cake," with the cylinders being the "layers" of the cake.



Complete this table for wedding cakes with each number of layers. Give answers in exact form.

	Number of layers	Lateral Area of cake	"Flat Area" of cake	Total Surface Area of cake	Volume of cake
1.	1				
2.	2				
3.	3				
4.	4				
5.	5				
6.	6				

Consider a wedding cake that has n layers. For each measure, write the letter of the expression in the list below at right that represents it. (Not all expressions will be used.)

7. lateral area of cake ______________

8. "flat area" of cake ______________

9. total surface area of cake ______________

10. volume of cake ______________

A. $n^3\pi$

B. $2n^2\pi$

C. $(n^2 + n)\pi$

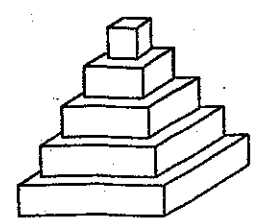
D. $(3n^2 + n)\pi$

E. $(3n^3 + 2n^2 + n)\pi$

F. $\frac{1}{6}(2n^3 + 3n^2 + n)\pi$

G. $\frac{4}{3}(n^3 + n^2 + n)\pi$

A figure that is similar to a wedding cake is the stack of right square prisms shown at right. The height of each prism in the stack is one unit. The bases of the prism at the top are squares with sides of length one unit. The bases of each other prism are squares with sides of length one unit greater than the bases of the prism directly above. An arrangement of prisms like this is sometimes called a "step pyramid," with the prisms being the "steps."



Consider a step pyramid that has n steps. Write an expression for each measure.

11. the total surface area of the pyramid

____

12. the volume of the pyramid

____

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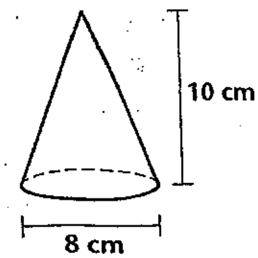
Enrichment

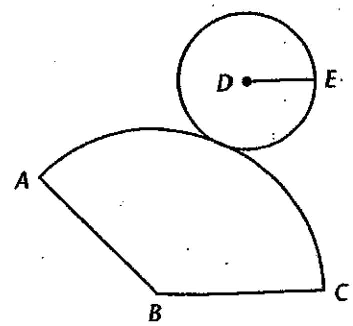
7.5 Making Nets for Right Cones

Suppose that you need to make a model of a cone that has the dimensions given in the figure at right. You know that the net for the cone consists of a circular region for the base and a region bounded by a sector of a circle for the lateral surface. But how do you know the exact size of each piece?



Give each measure for the right cone shown at right. When necessary, round to the nearest tenth of a centimeter.

1. radius ______________
2. circumference ______________
3. height ______________
4. slant height ______________
5. A sketch of a net for the cone shown above is given at right.



   a. Label the sketch with as many of the measures from Exercises 1–4 as possible.

   b. Suppose that you were to use the sketch to draw the net. Which important measure is still needed?

   ____

6. Refer to the net for the cone that you labeled in Exercise 5.

   a. Suppose that the *entire* circle with center at point B were drawn. What would be its circumference? ______________

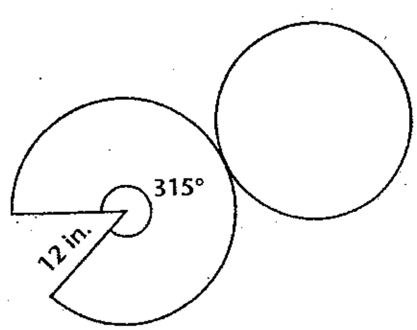
   b. What is the length of the arc that is drawn from A to C? ______________

   c. What percent of the entire circle is the arc from A to C? ______________

   d. Multiply 360° by your percent from part **c**. What is the measure of $\angle ABC$, rounded to the nearest whole degree? ______________

7. Refer to your results from Exercises 5 and 6. Using a compass, ruler, and protractor, draw an accurate real-size net for the cone. Then assemble the net to make a model of the cone.

8. A sketch of a net for a right cone is given at right. In the blank space to its right, draw the cone, making the height and diameter of the cone in the drawing proportional to the actual height and diameter. Be sure to label the height and diameter.



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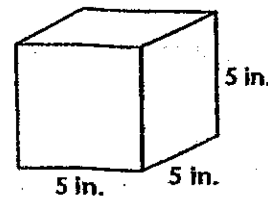
Enrichment

8.6 A "Reverse" Investigation of Area and Volume Ratios

Many real-life situations involving area and volume require you to perform a familiar process "in reverse" to achieve a desired result.



A manufacturer packages its product in a distinctive cubic package that has the dimensions shown at right. The manufacturer wants to offer the product in a cubic package that has twice the volume.

1. **a.** What is the volume of the original package? ______

   b. What is the volume of the new package? ______

   c. Write the ratio $\frac{\text{volume of new package}}{\text{volume of original package}}$ in simplest form. ______

2. **a.** What is the length of one edge of the original package? ______

   b. What is the length of one edge of the new package? (Hint: When you multiply a number by itself three times, you are *cubing* the number. What is the inverse operation?) ______

   c. Write the ratio $\frac{\text{length of edge of new package}}{\text{length of edge of original package}}$ in simplest form. ______

3. **a.** What is the surface area of the original package? ______

   b. What is the surface area of the new package? ______

   c. Write the ratio $\frac{\text{surface area of new package}}{\text{surface area of original package}}$ in simplest form. ______

Refer to the situation in Exercises 1–3. Suppose that the manufacturer wants to offer the product in a cubic package that has two-thirds the volume of the original. Write a ratio in simplest form to compare the given measure for the new package to the corresponding measure for the original package.

4. volume ______ 5. length of an edge ______ 6. surface area ______

7. Complete this statement: Two cubes with volumes in the ratio $\frac{a}{b}$ have linear measures in the ratio ______ and surface areas in the ratio ______.

8. Refer to the statement in Exercise 7. Conduct an investigation to determine whether the statement is true when the word *cubes* is replaced by the phrase *similar solids*. Show your work on a separate sheet of paper.

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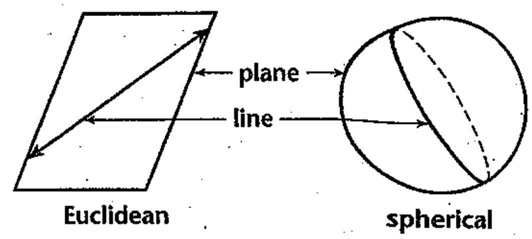
Enrichment

11.5 A Further Investigation of Spherical Geometry

In spherical geometry the meanings of the terms *plane* and *line* are different from their meanings in Euclidean geometry, as shown at right. As you have learned, a significant result is that the Parallel Postulates of the two geometries are very different. On this page you will investigate some other consequences of the differences between the meanings of these terms.



For Exercises 1 and 2, answer *yes* or *no*.

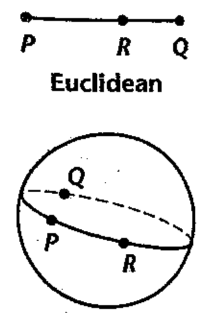
1. Does a line have endpoints:

a. in Euclidean geometry? __________ **b.** in spherical geometry? __________

2. Does a line have a measurable length:

a. in Euclidean geometry? __________ **b.** in spherical geometry? __________

3. In each figure at right, points *P*, *Q*, and *R* are collinear points positioned so that point *R* is between points *P* and *Q*.



a. In the Euclidean figure, what conclusion is drawn from the Segment Addition Postulate? __________

b. Can you draw the same conclusion about the spherical figure? Explain.

____

____

Each of the following statements is true in Euclidean geometry. On a separate sheet of paper, draw a figure to show why it is *false* in spherical geometry.

4. If two lines intersect, then their intersection is exactly one point.

5. One and only one line contains two given points.

6. Through a point outside a line, there is exactly one line perpendicular to it.

7. If two lines are perpendicular to the same line, then the two lines do not intersect.

Write two statements about relationships among points, lines, or planes that appear to be true both in Euclidean geometry and in spherical geometry.

8. ______________________________

9. ______________________________