Name___________________________________

Honors Algebra 2 — Ch 5 Notes Packet

Section 5.1: Transformations

Learning Target: We are learning to use transformations to graph quadratic functions.

Success Criteria:

- **I can transform quadratic functions**
- **I can describe the effects of changes in the coefficients of $\boldsymbol{y = a(x-h)^2} + k$**

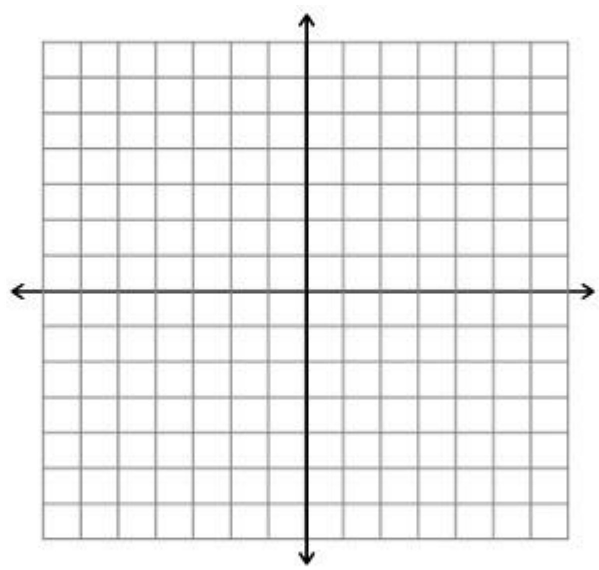
A _________________________________ is a function that can be written in the form $f(x) = ax^2 + bx + c$ or $f(x) = a(x-h)^2 + k$. Quadratic functions are easy to identify because the x variable is squared. The graph of a quadratic is also easy to identify—it's a

____

✓ Graphing a quadratic using a table is always an option. As you learn more about the properties of quadratics, you will develop other strategies to graph quadratics more efficiently.

Graph: $f(x) = x^2 - 4x + 3$ using a table

x	$f(x) = x^2 - 4x + 3$	$(x, f(x))$



Transformation of a function:

Parent Function vs Daughter Function

TRANSFORMATIONS SUMMARY		
Transformation	**General Form**	**Example**
Translation (Shift) Up	f(x) →	
Translation (Shift) Down	f(x) →	
Translation (Shift) Left	f(x) →	
Translation (Shift) Right	f(x) →	
Reflection (Flip) over x-axis	f(x) →	
Reflection (Flip) over y-axis	f(x) →	
Vertical Compression	f(x) →	
Vertical Stretch	f(x) →	
Horizontal Compression	f(x) →	
Horizontal Stretch	f(x) →	

Vertex Form: $f(x) = a(x-h)^2 + k$
where (h, k) = vertex

Ex1A: The parent function f(x) = x^3 is reflected over the x-axis, vertically stretched by a factor of 4, translated 3 units right and 1 units up. Write the new function g.

Ex1B: The parent function f(x) = x^2 is horizontally stretched by a factor of 5, translated 5 units down. Write the new function h.

Describe the transformation(s) that have been performed on the parent function $y = x^2$ to get the function given in each problem.

Ex2A: $g(x) = (\frac{1}{4}x)^2 + 2$

Ex2B: $h(x) = (2x + 6)^2 + 5$

What is the vertex of the daughter function?

What is the vertex of the daughter function?

Ex3A: The highway MPG, *m,* in miles per gallon for a compact car is approximated by the function $m(s) = -0.025s^2 + 2.45s - 30$, where *s* is the speed of the car in miles per hour. What is the maximum MPG for this car to the nearest tenth of a gallon? What speed results in this MPG?

You Try:

1. The parent function $f(x) = x^4$ is reflected over the x-axis, vertically compressed by a factor of $\frac{2}{3}$, translated 5 units left and 4 units up. Write the new function g.

2. Describe the transformation(s) that have been performed on the parent function $y = x^2$ to get the function given in each problem.

$y = \frac{1}{3}(x - 5)^2 - 2$

3. The average height *h* in centimeters of a certain type of wheat can be modeled by the function $h(r) = 0.024r^2 - 1.28r + 33.6$, where *r* is the distance in centimeters between the rows in which the wheat is planted. Based on this model, what is the minimum average height of the grain and what row spacing results in this height?

Section 5.2: Properties of Quadratics
Learning Target: We are learning about the properties of quadratic functions in all forms **Success Criteria:** • **I can define, identify, and graph quadratic functions.** • **I can identify and use maximums and minimums of quadratic functions to solve problems.**

A quadratic ____________________ is an equation that can be written in the form $ax^2 + bx + c = 0$ where a, b, and c are real numbers and a ≠ 0.

The _________________________________ of a quadratic function is the line through the vertex of a parabola that divides the parabola into 2 congruent halves.

The _________________________ of a graphed quadratic function is any point(s) where the graph crosses the x-axis.

The _________________________ of a graphed quadratic function is any point where the graph crosses the y-axis.

Intercept Form:	**Vertex Form:**	**Standard Form:**

- ✓ All graphs of quadratic functions are ___________________, have _______________________, and a lowest/highest point called the ___________________ or ____________________.
- ✓ The left and right ends of the parabola go in the same direction (either both go infinitely up or both go infinitely down)
- ✓ The sign on the coefficient of x^2 (the "a value") determines if the parabola opens up or down.
- ✓ Key parts of a quadratic function's graph are: vertex, AOS, end behavior, y-int, and x-ints (if they exist)

To find:	Intercept form	Vertex form	Standard form
Direction			
AOS			
Vertex			
Y-int			
X-int(s)			

What is the DOMAIN and RANGE?

Example of: NOTATION — HOW TO READ NOTATION

Let $f(x) = x^2$

	NOTATION	HOW TO READ NOTATION
Domain:	$\{x \mid x \in \mathbb{R}\}$	"The set of all numbers x such that_____________ ____"
Range:	$\{y \mid y \geq 0\}$	"The set of all numbers y such that_____________ ____"

Ex 1: Find the vertex, direction parabola opens, AOS, y-int, x-ints and domain and range.

A. $y = (-x - 5)(x + 2)$ **B.** $y = (x + 1)^2 - 6$ **C.** $y = -4x^2 - 8x + 5$

Ex2: Find the max/ min of the function. Then state its domain and range: $g(x) = -5x^2 + 15x - 3$

APPLICATIONS:

Ex3: A community theatre sells about 100 tickets to a play each week when it charges $20 per ticket. For each price decrease of $2, about 20 more tickets per week are sold. The theatre has fixed expenses of $750 per week. Write a verbal equation for the theatre's profit for the week. Then write an algebraic equation for profit P(x). Use tables/graph to find how the theatre can maximize profit.

You Try:

1. Find the vertex, direction parabola opens, AOS, y-int, x-ints and domain and range for:

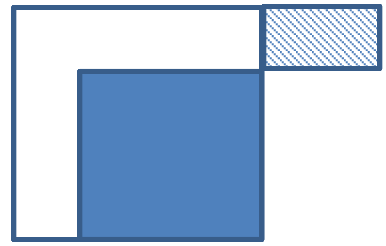
A. $y = -2(x-5)^2 + 8$

B. $y = x^2 - 8x - 48$

2. Find the max/min and state the domain and range of the function: $h(x) = x^2 - 4x - 5$

3. A go-cart track has about 380 racers per week and charges each racer $35 to race. The owner estimates that there will be 20 more racers per week for every $1 reduction in price per racer. Write an equation for the revenue R(x) that the owner could use to determine his expected revenue, supposing x represents the price reduction. What is the maximum revenue? When did this occur?

4. Jerry is spreading a new layer of garden soil on his garden. He decided that he wants to expand his garden by the same length on 2 sides of the garden. He also wants to spread some soil in a small 12 ft^2 are off the northeast edge of his garden. If the garden was originally 8 ft by 8 ft, write an equation for the area A(x) he is covering with garden soil.



Section 5.3: Solving Quadratics by Graphing, Simple Solving & Factoring
Learning Target: We are learning how to solve for the solutions/ zeros/ roots of a quadratic equations. **Success Criteria:** • **I can solve quadratic equations by graphing or factoring.** • **I can determine a quadratic function from its roots.**

The ___________________ of a function is a value of the input x that makes the output f(x) = 0.

Similarly, they may also be called: _______________________, _______________________ &

____

Look back at Warm-up 5.3:

5 Main Ways to Find the ZEROS (or)

1-

2-

3-

4-

5-

SIMPLE SOLVING:

Ex 1: Find the solution(s).

A. $(x-4)^2 = 24$

B. $(x+32)^2 + 6 = 175$

C. $x^2 - 16x + 63 = 0$

FACTORING: Remember, if a•b = 0 then ___________________________________

First step in factoring =

Otherwise:

Ex 2: Find the zeros of the quadratic function by factoring.

A. $f(x) = 4x^2 - 20x$

B. $f(x) = 9x^2 + 3x - 56$

Special Factoring Rules	

Ex 3: Find the roots of the equation by factoring.

A. $25x^2 - 81 = 0$

B. $5x^2 + 20 = 20x$

Ex 4: Write the simplest standard form function with integer coefficients that can be written from the given zeros.

A. x = 0, 1, – 4

B. x = ½ , – 3

Ex 5: The product of 2 consecutive odd integers is 195. Use quadratics to solve for the integers.

VERTICAL MOTION MODEL (aka Projectile Motion):

$$\boldsymbol{h(t) = -16t^2 + v_0 t + h_0}$$

$v_0 =$

- $+v_0$ *means*:
- $-v_0$ *means*:
- *no* v_0 *means*:

Ex 6:

A. A penny is dropped from a bridge 30 feet above a wishing pool on the ground. How long does it take the penny to reach the pool?

B. A toy rocket is launched upward from the ground at an initial velocity of 80 ft/s.

- How high will the rocket be after 2 seconds?
- At what time(s) will the rocket be 36 ft?
- What is the highest point of the rocket?

You Try:

1. Find the solution(s).

A. $x^2 + 5 = 30$

B. $(x + 115)^2 + 16 = 641$

2. Find the zeros of the quadratic function by factoring.

$f(x) = 2x^3 + 8x^2 - 42x$

3. Find the roots of the equation by factoring.

$4x^2 + 28x = -49$

4. Write the simplest standard form function with integer coefficients that can be written from the given zeros

$x = -2, \frac{1}{5}$

5. My 108 square foot family room is 3 feet shorter in length than it is in width. Find the dimensions of my family room using quadratics.

6. Fireworks are shot into the air from the ground with an initial velocity of 192 ft/s.

A. How long does it take for the fireworks to reach their maximum height?

B. What is the maximum height?

Section 5.4: Completing the Square
Learning Target: We are learning how to solve for the solutions/ zeros/ roots of a quadratic equations. **Success Criteria:** • **I can solve quadratic equations by completing the square.** • **I can write quadratic equations in vertex form.**

Solving Quadratics: Determine which method of solving you feel would be the most efficient way to solve the problems below. Explain briefly why you believe this method to be the most efficient. Then solve:

1. Method chosen & explanation why?

$(x + 3)^2 = 2$

2. Method chosen & explanation why?

$x^2 + 6x + 7 = 0$

Is one of the problems given more difficult than the other? Why/ in what way?

Look at both equations graphs on your graphing calculator: What do you notice?

If one form of a quadratic equation doesn't "work" for us, it may be helpful to manipulate it into another form that does! Let's explore HOW to do that by "*<u>COMPLETING THE SQUARE</u>*":

So how about…

1. $x^2 + 8x +$ ________

2. $x^2 + 20x +$ ________

What is the pattern for completing the square when a = 1?

$x^2 + bx +$ ______, when a = 1

Ex1: Complete the square. Then rewrite as a binomial squared.

A. $x^2 - 20x +$ ________

B. $x^2 + 5x +$ ________

Ex 2: Solve the quadratic by completing the square.

A. $x^2 - 6x + 7 = 0$

B. $2x^2 + 5x - 3 = 0$

Even if you are not solving for the roots, you can use the completing the square strategy (with a little twist) to manipulate a quadratic from a standard form equation to a vertex form equation.

Ex3: Write the function in vertex form. Then identify the vertex.

A. $f(x) = x^2 - 14x + 34$

B. $f(x) = 3x^2 + 18x + 15$

You Try:

1. Complete the square: $x^2 - 9x +$ ___________

2. Solve the quadratic by completing the square: $2x^2 - 8x + 14 = 0$

3. Write the function in vertex form. Then identify the vertex: $f(x) = 2x^2 + 6x - 10 = 0$

Section 5.6: Quadratic Formula
Learning Target: We are learning how to solve for the solutions/ zeros/ roots of a quadratic equations. **Success Criteria:** • **I can solve quadratic equations by using the Quadratic Formula.** • **I can classify roots using the discriminant.**

For quadratic equations in the form of $ax^2 + bx + c = 0$, what is the quadratic formula?

Define the discriminant:

What does it tell you about the solutions of a quadratic equation? (explain all three cases clearly)

-
-
-

Ex1: Classify the roots of $f(x) = 4x^2 - 7x + 2$ using the discriminant, then solve for the roots of f(x).

Section 5.5: Complex Numbers

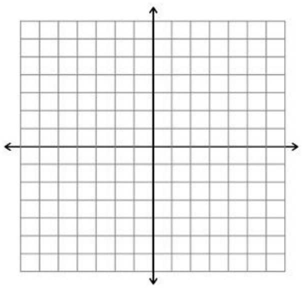
Learning Target: We are learning about imaginary numbers, complex numbers and complex roots of a quadratic equations.

Success Criteria:

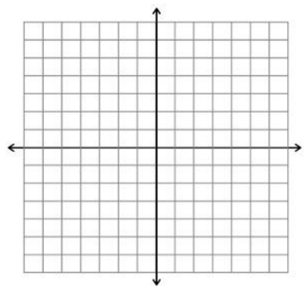
- **I can define and use imaginary and complex numbers.**
- **I can solve quadratic equations with complex roots.**

Recall the discriminant $b^2 - 4ac$ is either +, − , or 0. Sketch a function illustrating each case.

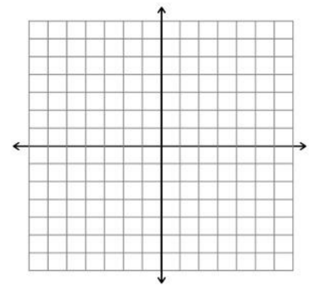
$b^2 - 4ac > 0$



$b^2 - 4ac = 0$



$b^2 - 4ac < 0$



The last scenario puzzled mathematicians for a very long time. In the 1500's, Italian mathematician Girolamo Cardona was the first to use complex numbers. In the 1700's, French mathematician Leonhard Euler defined the imaginary unit. Later in the 1800's, the theoretical basis of complex numbers was rigorously developed.

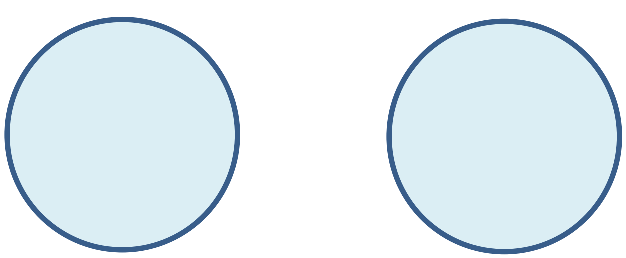
The imaginary unit is i.

i =

An imaginary number is the ______________________________ of any __________________ number and is written as:

The square of an imaginary number is the original ________________________ number.

An complex number is any __________________ number that can be written in the form where a and b are real numbers and i is the imaginary unit.



Complex

Real **Imaginary**

Rational **Irrational**

Ex1: Simplify each in terms of i

A. $\sqrt{-484}$

B. $\sqrt{-96}$

Complex numbers are equal when:

Ex2: Find the value of x & y so that the complex numbers are equal: $4x + 10i = 2 - 4yi$

Complex Conjugate Pairs:

Ex3: Write the complex conjugate

A. $8 - 5i$

B. $15i - 7$

C. $-6i$

Ex4: Find the zeros of the function f(x)

A. $f(x) = x^2 + 4$

B. $f(x) = (x - 5)^2 + 12$

You Try:

1. Simplify: $\sqrt{-450}$

2. Write the complex conjugate: $-18 - 3i$

3. Find the zeros of the function: $f(x) = 2(x - 7)^2 + 18$

Section 5. 9: Operations with Complex Numbers

Learning Target: We are learning about operations involving complex numbers.

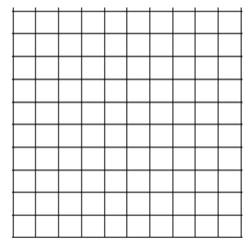
Success Criteria:

- **I can perform operations on complex numbers.**

A COMPLEX PLANE has a real axis and an imaginary axis.

Ex 1: Graph the points on the complex plane

A = 2 + 3i
B = -1 + 4i
C = 4 – i
D = -5i



The ABSOLUTE VALUE of a COMPLEX number [notation:]:

Ex2: Find the absolute value of the complex number

A. |3 + 5i| **B.** |-7i|

ADDING & SUBTRACTING COMPLEX NUMBERS:

Ex3:

A. (4 + 2i) + (– 7i + 6) **B.** (5 – 2i) – (– 2 – 3i)

MULTIPLYING & DIVIDING COMPLEX NUMBERS:

Ex4:

A. $-3i^{35} \bullet 4i^{10}$

B. $(3 + 6i)(4 - i)$

C. $\dfrac{(3+10i)}{5i}$

D. $\dfrac{(2+8i)}{(4-2i)}$

You Try:

1. Find $|-13 + 5i|$

2. Simplify: $(8 + 5i) - (8 - 4i)$

3. Simplify: $(2-9i)(2+9i)$

4. Simplify: $\dfrac{(9+4i)}{(2-i)}$

Section 5.8: Curve Fitting with Quadratic Models
Learning Target: We are learning about creating quadratic models from data points. **Success Criteria:** • **I can use quadratic functions to model data.** • **I can use quadratic models to analyze and predict.**

How do you determine whether data is quadratic using the data in tables?
Investigate the patterns of each of the tables below:

x	y
-2	-7
-1	0
0	5
1	8
2	9
3	8

x	y
-2	-7
-1	-3
0	-1
1	5
2	9
3	13

x	y
-2	-15
-1	-2
0	5
1	6
2	1
3	-10

Creating a Quadratic Regression Equation	
To Enter Data	**To Make Equation**
STAT Edit → 1 Edit enter x values in the 1st list (L1) and y values in the 2nd list (L2)	STAT → CALC ↓ Quad Reg L1, L2
To Graph Data	**To Enter Regression Equation into y =**
2nd then y = (Stat Plot) Enter → On → Enter ↓ (dotted curve) → Enter Use x and y lists as entered & choose mark Adjust WINDOW to fit data: Change X-MIN and X-MAX to fit x data Change Y-MIN and Y-MAX to fit y data	Create Regression Equation as above Y = (clear away any other equations) VARS ↓ STATISTICS → EQ REG EQ → Enter

Ex1: The table shows the cost of circular plastic wading pools based on the pools diameter.

Diameter (ft)	Cost
4	\$19.95
5	\$20.25
6	\$25.00
7	\$34.95

Find a quadratic model for the cost of a wading pool given its diameter.

What would be the cost of a pool with an 8ft diameter?

A 10 foot diameter?

Ex2: The table shows the approximate run times and reel length of a 16mm film given the diameter of the film on the reel.

Diameter (in)	Reel Length (ft)	Run Time (min)
5	200	5.55
7	400	11.12
9.25	600	16.67
10.5	800	22.22
12.25	1200	33.33
13.75	1600	44.45

Find a quadratic model for the run time given the diameter.

Estimate the run time for a reel of film with a diameter of 15 in.

Find a quadratic model for the reel length given the diameter.

Estimate the reel length for a reel of film with a diameter of 8 in.

Creating the Standard Form of a Quadratic from Points (WITHOUT a calculator):

Let's use 3 points from the first table at the beginning of this section (   ), (   ), (   ).
Be STRATEGIC in your selection of points!

Let's try another: (   ), (   ), (   )

You Try:

1.

Common US Copper Wire Gauges	
Gauge	Diameter (in)
24	.0201
22	.0254
20	.0320
18	.0403

A. Find the quadratic regression to model the diameter given the wire gauge.

B. Use your model to predict the diameter for a 12 gauge copper wire.

2. Write a quadratic function that fits the data without using a calculator.
(2,3) (6, 3) (8, -3)

Section 5.7: Solving Quadratic Inequalities

Learning Target: We are learning about graphing and solving quadratic inequalities.

Success Criteria:

- **I can graph quadratic inequalities.**
- **I can solve quadratic inequalities using graphs and tables.**
- **I can solve quadratic inequalities by using algebra.**

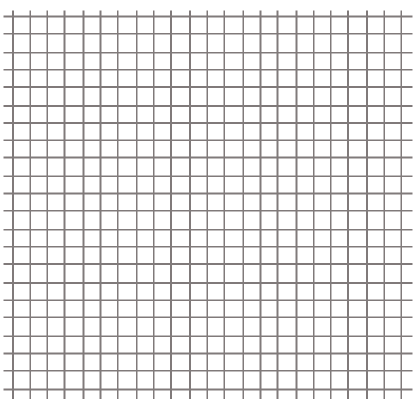
<u>Graphing</u> Quadratic Inequalities (note: 2 variables for graphing)

1- Graph parabola defined by the boundary (include important points: vertex, intercepts, symmetric points, etc.)
2- Solid line for ≥ ≤. Dashed line for > <.
3- Shade above parabola for y ≥ or y >. Shade below parabola for y ≤ or y <. [This only works if the y is alone on the **<u>left</u>** side. If not, you will need to test points to find the shaded region]

Ex 1:

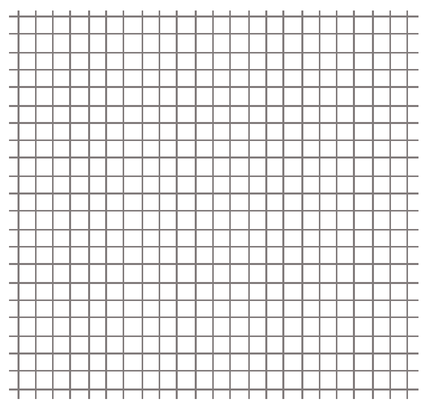
A. Graph $y > x^2 - 7x + 10$

Show work to find important features of the quadratic below. Report results in an organized way. Then graph on the plane.



B. Graph $y \le 2x^2 - 4x - 1$

Show work to find important features of the quadratic below. Report results in an organized way. Then graph on the plane.



<u>Solving</u> Quadratic Inequalities (note: 1 variable for solving- y has been replaced with 0)

What are you trying to find?

Critical values:

Solving Quadratic Inequalities is usually done by-

using tables and graphs (in calc) OR using algebraic solving

Ex2: Solve the quadratic inequality using tables and graphs

A. $x^2 + 8x + 20 \geq 5$

B. $2x^2 - 5x + 1 < 1$

Ex3: Solve the quadratic inequality algebraically

A. $x^2 - 10x + 18 \geq -3$

B. $-2x^2 + 3x + 7 < 2$

Ex4: The monthly profit of a small business that sells bike helmets can be modeled by the function $P(x) = -8x^2 + 600x - 4200$, where x is the average selling price of a helmet. What range of selling prices will generate a monthly profit of at least \$6000?

You Try Answers:

Sec 5.1:

1. $g(x) = -\frac{2}{3}(x+5)^4 + 4$
2. Vertical compression by $\frac{1}{3}$
   Translation 5 right
   Translation 2 down
3. Avg height = 16.5 cm
   Row spacing = 26.7 cm

Sec 5.2

1A. vertex: (5, 8)
   Opens down
   AOS: x = 5
   y-int: -42
   x-ints: 3 & 7
   D: $\{x| \; x \in \mathbb{R}\}$
   R: $\{y| \; y \le 8\}$

1B. vertex: (4,-64)
   Opens up
   AOS: x = 4
   y-int: -48
   x-ints: -4 & 12
   D: $\{x| \; x \in \mathbb{R}\}$
   R: $\{y| \; y \ge -64\}$

2. minimum (2, 7)

   D: $\{x| \; x \in \mathbb{R}\}$
   R: $\{y| \; y \ge 7\}$

3. Max revenue of $14,580 occurs after 8 price reductions so the cost for each racer is $27

4. A(x) = x^2 + 16x + 78

Sec 5.3:

1A. $x = \pm 5$
1B. x = -90 or -140
2. x = -3, 0, 7
3. x = $-\frac{7}{2}$
4. f(x) = $5x^2 + 9x - 2$

Sec 5.4:

1. $8\frac{1}{4}$
2. $x = 2 \pm \sqrt{11}$
3. $f(x) = 2\left(x + 1\frac{1}{2}\right)^2 - 14\frac{1}{2}$
   Vertex $(1\frac{1}{2}, -14\frac{1}{2})$

Sec 5.6:

None

Sec 5.5:

1. $3i\sqrt{5}$
2. -18 +3i
3. $x = 7 \pm 3i$

Sec 5.9:

1. $\sqrt{194}$
2. 9i
3. 85
4. $\frac{14+17i}{5}$

Sec 5.8

1A. y = $.0001875x^2$ - .011235x + .18176
1B. .0739 inches
2. $f(x) = -\frac{1}{2}x^2 + 4x - 3$

Sec 5.7

None