Name______________________________________

Honors Algebra 2 — Ch 7 Notes Packet: Exponentials & Logarithms

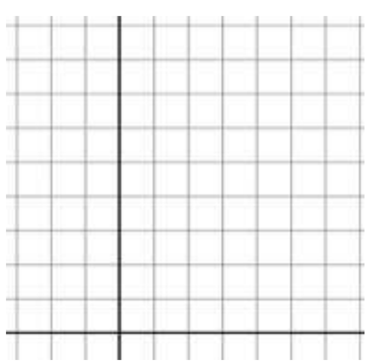
Section 7.1: Exponential Growth and Decay Functions
Learning Target: We are learning about exponential growth and decay **Success Criteria:** • **I can write and evaluate exponential expressions to model exponential growth and decay.**

Define and give the general form (label all parts) of an EXPONENTIAL FUNCTION:

Make a table for f(x) and graph:

$f(x) = 2^x$

x	-2	-1	0	1	2	3
y						



Complete the following:

Asymptote: y =

Domain: { x | }

Range: { y | }

End behavior: $as\ x \to -\infty,\ f(x) \to$

$as\ x \to +\infty,\ f(x) \to$

Define the following terms-

Asymptote:

Exponential decay:

Exponential growth:

Write and label the general formula for a constant percent increase/ decrease exponential:	Write and label the general formula for a compound interest exponential formula: (look in HW pages for the equation)

****STOP: Complete WS 7.1 Practice Problems. Then continue…***

<u>After you have completed the Practice Problems, answer the following:</u>

What is the difference between a ***decay*** factor and a ***growth*** factor?	What is the difference between a decay ***factor***, decay ***rate*** *and decay* ***percent***?	How do you incorporate "doubling" or "tripling" or "halving" into an exponential function?

Section 7.2: Inverses of Relations and Functions
Learning Target: We are learning about inverses of relations and functions. **Success Criteria:** • **I can graph and recognize inverses of relations and functions.** • **I can find inverses of functions.**

What is the inverse of ADDITION?	**What is the inverse operation of MULTIPLYING?**	**What will undo SQUARING?**
What is the inverse of SITTING DOWN?	**What is the inverse of TURNING RIGHT?**	**What is the inverse of SPENDING?**

Directions to Dave & Buster's:	Directions back to NHS from Dave & Buster's:
• Turn left out of NHS parking onto 6 Mile Rd • Turn left onto Haggerty Rd • Turn right onto 7 Mile Rd • Turn left onto Victor Parkway	

Relations:

Functions:

- **Inverse Relation:**

- **Inverse Function:**

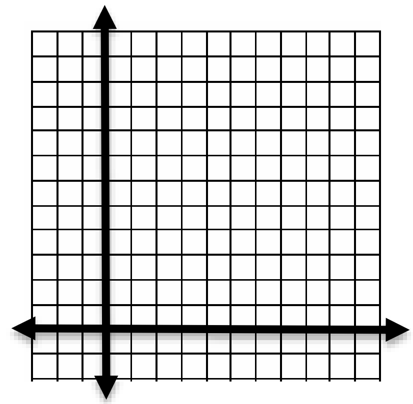
Notation:

Ex 1: Examine the inverse functions: $f(x) = \sqrt[3]{\boldsymbol{x}} + \mathbf{1}$ & $\boldsymbol{f^{-1}(x)} = (\boldsymbol{x} - \mathbf{1})^3$

Ex 2: Given the relation below, graph the relation and the inverse of the relation. Then identify the domain and range of the original relation and the inverse of the relation.

x	y
0	3
1	6
4	9
6	10
10	11

x	y



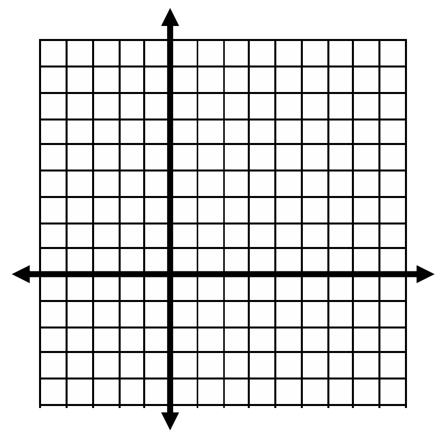
An inverse of a function ______________________________________

***** If (a, b) is on f(x), then ____*****

Ex 3: Graph the relation and its inverse. Examine the graph.

x	y
− 2	$\frac{1}{4}$
− 1	$\frac{1}{2}$
0	1
1	2
2	4

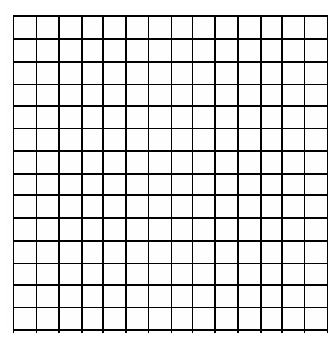
x	y



The graph of an inverse of a function is ______________________________________

Ex 4: Graph the function. Then use inverse operations to write and graph its inverse.

A. $f(x) = 2x - 6$



B. $f(x) = \frac{2}{3}x - 4$

Ex 5: Application: Juan buys a iPhone case online for 20% off the list price. He has to pay $2.50 for shipping.

A. Write an equation for the list price in terms of the cost of the iPhone case.

B. What is the list price of the CD if his cost is $13.70?

<u>You Try</u>

1. Graph the function. Then use inverse operations to write & graph its inverse. $f(x) = -3x - 6$

2. Graph the function. Then use inverse operations to write & graph its inverse. $f(x) = \frac{3x+1}{2}$

Section 7.3 Logarithmic Functions
Learning Target: We are learning about logarithmic functions **Success Criteria:** • **I can write equivalent forms for exponential and logarithmic functions.** • **I can write, evaluate, and graph logarithmic functions.**

Logarithms are used to find unknown exponents in exponential models. Logarithmic functions define many measurement scales in science such as pH, decibels and Richter scale.

Exponential Form ***Logarithmic Form***

Exponent

$2^5 = 32$ $\log_2 32 = 5$

Base

Ex1: Rewrite the Warm-Up problems as logarithms.

A. $2^x = 16$ B. $3^x = 81$ C. $5^x = 125$ D. $2^x = 1024$ E. $10^x = 2580$

Ex2: Write each exponential equation in logarithmic form or vice versa.

A. $25^{1/2} = 5$ B. $\log_7 49 = 2$ C. $6^{-1} = \frac{1}{6}$ D*. $\log_{10} 10 = 1$ E*. $\log_5 1 = 0$

SPECIAL LOGARITHM PROPERTIES:

PROPERTY	**because**	**Example:**
1-		
2-		

***** if no base of log is written, it is understood to be a log with base _____________**

Ex3: Use mental math to evaluate.

A. $\log_4 64$ B. $\log\ .01$ C. $\log_5 \left(\frac{1}{5}\right)$ D. $\log_3 \left(\frac{1}{9}\right)$

- The logarithm with base 10 is called the ___________________________________. It is denoted either **log** $_{\mathbf{10}}$ or simply **log**.
- The logarithm with a base e is called the ___________________________________. It can be denoted either **log** $_{\mathbf{e}}$ but is usually denoted **ln**.

GRAPHING EXPONENTIALS & LOGS [Remember the important facts about functions that are inverses]

Ex4:

A. Make a table and graph:

$f(x) = 2^x$

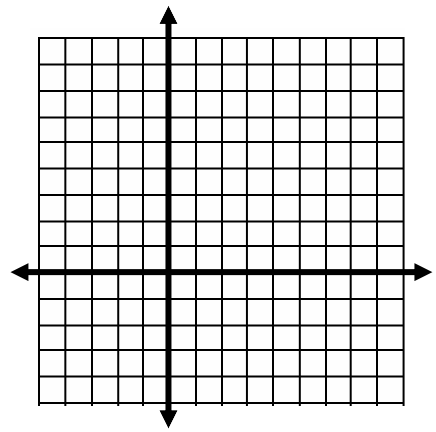
Domain:

Range:

B. Find $f^{-1}(x)$, make a table and graph:

Domain:

Range:



APPLICATION: Acidity is measured in pH which is linked to the concentration of hydrogen ions [H^+] measured in moles per liter by the equation: pH = -log[H^+]

Complete the table:

Substance	**[H^+] concentration (mol/L)**	**pH**
Milk	0.00000025	
Tomatoes	0.0000316	
Lemon Juice	0.0063	
Iced Tea		3.8
Ginger Water		7.21

Section 7.4: Properties of Logarithms
Learning Target: We are learning about the properties of logarithms **Success Criteria:** • **I can use properties to simplify logarithmic expressions.** • **I can convert between logarithms in any base.**

Just like exponents have special properties, so do logarithms. Recall the special exponent properties:

Product: $a^m \cdot a^n = a^{m+n}$ Quotient: $\frac{a^m}{a^n} = a^{m-n}$ Power: $(a^m)^n = a^{mn}$

Each property of exponents has a corresponding property of logarithms. In addition, there are inverse properties and a way to change the base of logarithms.

Product Property of Logarithms: For any positive numbers m, n, and b (b ≠ 1), the logarithm of a product is equal to the sum of the logarithm of its factors. Property: **Quotient Property of Logarithms:** For any positive numbers m, n, and b (b ≠ 1), the logarithm of a quotient is equal to the log of the dividend minus the logarithm of the divisor. Property: **Power Property of Logarithms:** For any real number p and positive numbers a & b (b ≠ 1), the logarithm of a power is the product of exponent and the logarithm of its base. Property:	**Inverse Property of Logarithms and Exponents:** For any base b such that b > 0 and b ≠ 1 Property: Property: **Change of Base Formula:** For a > 0 and a ≠ 1 and any base such that b > 0 and b ≠ 1 Property: Example:

Examples:

APPLICATION:
Seismologists use the Richter scale to express the energy/ magnitude of an earthquake. The Richter magnitude, M, of an earthquake is related to the energy released in ergs, E, by the formula:

$$M = \frac{2}{3}\log\left(\frac{E}{10^{11.8}}\right)$$

Ex5: A tsunami that devastated parts of Asia in December 2004 was spawned by an earthquake with magnitude 9.3. How many times as much energy did this earthquake release compared to the 6.9 magnitude earthquake that struck San Francisco in 1989?

Section 7.5: Exponential and Logarithmic Equations & Inequalities
Learning Target: We are learning about exponential and logarithmic equations and inequalities **Success Criteria:** • **I can solve exponential and logarithmic equations and inequalities.** • **I can solve problems involving exponential and logarithmic equations.**

Exponential Equation:

Logarithmic Equation:

Exponential/ Logarithmic Inequality:

Hints to Solve Exponentials and Logarithms:

<u>For exponentials:</u>
- Try to get each side of the equation to have the same base
- Take the log of both sides to solve exponentials

<u>For logarithms:</u>
- Try to get the same base log on each side of the equation
- Simplify using log properties
- Exponentiate when you have one log

If not sure, try to convert to the other form to see if that helps trigger a solving option

Examples: *FYI -textbook has some alternative methods to solving some of these--they can be done different ways!

Ex1: Solve algebraically. Check your answers with your calculator.

A. $3^{2x} = 27$ B. $(\frac{1}{2})^{4x+1} = (8)^{2x+1}$ *C. $\log_3 (x - 5) = 2$ D. $\log_2 n = \frac{1}{3}\log_2 27 + \log_2 36$

Ex2: Use your calculator's graph and table functions to solve.

A. $2^{2x} = 1024$ B. $\log x - \log 2 = \log 75$ C. $\log x^2 < 6$

✎You Try… see You Try handout.

Section 7.6: Natural Base, e
Learning Target: We are learning about the natural base **Success Criteria:** • **I can use the number *e* to write and graph exponential functions representing real-world situations.** • **I can solve equations and problems involving *e* or natural logarithms.**

Suppose you move to the "big city" after college and see a bank advertisement that offers: 100% INTEREST ON ALL SAVINGS ACCOUNTS! The teller you approach pulls out a stack of papers and asks you what kind of account you'd like to open – one compounded yearly, quarterly, monthly, weekly, daily, hourly, minutely or … continuously. You've hear of all the others, but you don't know what he means by continuously, so you ask him. He replies that the conditions of the offer don't allow you to tell him what it means. Actually, the only way you can get an account here is to be clever enough to explain to him what it means. He makes you a deal. If you can tell him how much money a single dollar would be worth after 1 year at each of the other rates, you may be able to make a guess at what kind of interest continuous interest is. If you can do that, the account is yours at 100% interest. He will even give you the compound interest formula: $A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$ but insists that you should recall or find out what each piece of the formula means.

- Use the Compound Interest Formula to find the amount in the account after one year if you deposited $1 in the account.

- Complete the table.

Compounding frequency	**n**	**Amount after 1 year**
Annually		
Semiannually		
Quarterly		
Monthly		
Daily		
Hourly		
Every minute		

- As n increases the situation approaches what banks call continuous compounding. What amount does the function seem to approach?

In the 100% interest bank situation we noted that as n gets bigger and bigger, the output of $A(t) = P\left(1 + \frac{1}{n}\right)^t$ approaches ≈

- ✓ This number has a special notation: ___________ sometimes referred to Euler's number or Napier's constant, is an irrational number like $\sqrt{3}$ and π and can be used as a base of an exponential just like any other number. The properties of exponents work exactly as they would for any other base.

- ✓ Similarly, you can convert exponentials involving e to logarithms. When you have a log with base e it is called a ___________________________ and has the notation _________ instead of $\log_e$. All log properties and rules apply including change of base.

Ex 1: Simplify

A. B. C. D. E.

When graphing natural base exponential functions in the form $f(x) = a \cdot e^{rx}$ if a > 0 and r > 0 then it is a growth function. If a > 0 and r < 0 then it is a decay function.

Ex 2: Is the function growth or decay?

A. $f(x) = \frac{1}{8} \cdot e^{5x}$ B. $f(x) = 2e^{-8x}$

Ex 3: Graph f(x) on your calculator. Determine the domain and range and if it is growth or decay. Then write the equation for and graph the inverse function.

A. $f(x) = e^x$

B. $f(x) = e^{.62x} + 2$.

Continuous compounding equation: $\mathbf{A = Pe^{rt}}$

Ex2:

A. What is the total amount for an investment of $100 invested at 3.5% for 8 years compounded continuously?

B. What is the total amount for an investment of $500 invested at 5.25% for 40 years compounded continuously?

e is also used in real life situations dealing with radioactive decay and half-life. **What is half-life? Explain what is occurring…**

Half Life Equation: $N(t) = N_0e^{-kt}$

Ex3:

A. Determine how long it will take for 650 mg of chromium-51 which has a half-life of about 28 days to decay to 200mg.

B. Plutonium-239 has a half-life of 24,110 years how long does it take for a 1 g sample to decay to 0.1 g?

Section 7.7: Transformations of Exponential and Logarithmic Functions

Learning Target: We are learning about transformations of exponential and logarithmic functions.

Success Criteria:
- **I can transform exponential and logarithmic functions by changing parameters.**
- **I can describe the effects of changes in the coefficients of exponential and logarithmic functions.**

Reading Strategies 7.7, then:

	Parent Function →	**$f(x) = 2^x$**	**$f(x) = \log(x)$**
Transformation	**f(x) notation**	**Exponential Examples**	**Logarithmic Examples**
Vertical Translation			
Horizontal Translation			
Vertical Stretch/ Compression			
Horizontal Stretch/ Compress			
Reflection			

Write the equation of the daughter function transformed from the given parent function.

Ex 1: Parent function: $f(x) = 3^x$

A.

- Reflection across the x-axis
- Shift up 1
- Then vertical stretch by 2

B.

- Reflection across the x-axis
- Vertical stretch by 2
- Then shift up 1

C. Graph Ex1 parent f(x) and A & B on calc, then find each domain and range

D:
R:

D:
R:

D:
R:

<u>**You Try:**</u>

1. Write the equation for parent function: $f(x) = e^x$

A.

- Vertical compression by ½
- Shift up 4, then
- Reflection across the x-axis

B.

- Shift left 2
- Shift up 4, then
- Vertical compression by ½

2. Write the equation for parent function: f(x) = log x

A.

- Reflection across the x-axis
- Shift up 1
- Then, shift right 3

B.

- Reflection across the y-axis
- Horizontal stretch by 2
- Then shift up 4

Section 7.8: Curve Fitting with Exponential and Logarithmic Functions
Learning Target: We are learning about creating equations to model exponential and logarithmic data. **Success Criteria:** • **I can model data by using exponential and logarithmic functions.** • **I can use exponential and logarithmic models to analyze and predict.**

How do you know if data is:

- Linear?
- Quadratic?
- Cubic?
- Exponential?

x	y
-1	2
0	3
1	5
2	8
3	12

x	y
-1	16
0	24
1	36
2	54
3	81

As with other functions, we can create regression equations using exponential or logarithmic data.

Ex1: Determine whether f is an exponential function of x. If so, find the constant ratio.

A.

x	–1	0	1	2	3
$f(x)$	$\frac{5}{6}$	$\frac{5}{2}$	7.5	22.5	67.5

B.

x	–1	0	1	2	3
$f(x)$	1	0.5	0.33	0.25	0.2

Ex2: Bernice is selling seashells found at the beach. The price of a shell depends on its length.

Length of Shell (cm)	5	8	12	20	25
Price ($)	2	3.5	5	18	40

Find an exponential model for the data.

What is the length of a shell selling for $9.00?

If Bernice found a 40 cm shell. How much could she sell it for?

Ex3:

Time (min)	1	2	3	4	5
Speed (m/s)	1.5	6.2	10.6	12.9	14.8

Use logarithmic regression to find a function that models this data.

When will the speed exceed 20 m/s?

What will the speed be after 1 hour?

You Try:

1. Find a natural log model for the data:

Global Population Growth	
Population in billions	**Year**
1	1800
2	1927
3	1960
4	1974
5	1987
6	1999

According to the model, when will the global population exceed 9,000,000,000?

<u>You Try answers:</u>

<u>Sec 7.2:</u>

1. $f^{-1}(x) = -\frac{1}{3}x - 2$
2. $f^{-1}(x) = \frac{2}{3}x - \frac{1}{3}$

<u>Sec 7.7:</u>

1. A. $g(x) = -\frac{1}{2}e^{x} + 4$
   B. $g(x) = \frac{1}{2}e^{x+2} + 2$
2. A. $g(x) = -\log(x - 3) + 1$
   B. $g(x) = \log(-\frac{1}{2}x) + 4$

Sec 7.8:

1. 1824.41 + 106.48lnx
   In the year 2058