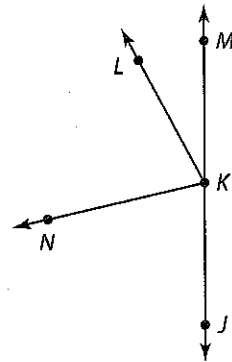


Example 3 Measure and Classify Angles

ALGEBRA In the figure, \overrightarrow{KJ} and \overrightarrow{KM} are opposite rays, and \overrightarrow{KN} bisects $\angle JKL$. If $m\angle JKN = 8x - 13$ and $m\angle NKL = 6x + 11$, find $m\angle JKN$.



Step 1 Solve for x .

Since \overrightarrow{KN} bisects $\angle JKL$, $\angle JKN \cong \angle NKL$.

$m\angle JKN = m\angle NKL$ Definition of congruent angles

$8x - 13 = 6x + 11$ Substitution

$8x = 6x + 24$ Add 13 to each side.

$2x = 24$ Subtract $6x$ from each side.

$x = 12$ Divide each side by 2.

Step 2 Use the value of x to find $m\angle JKN$.

$m\angle JKN = 8x - 13$ Given

$= 8(12) - 13$ $x = 12$

$= 96 - 13$ or 83 Simplify.

Guided Practice

3. Suppose $m\angle JKL = 9y + 15$ and $m\angle JKN = 5y + 2$. Find $m\angle JKL$.

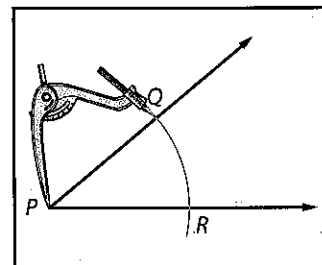
Study Tip

Checking Solutions Check that you have computed the value of x correctly by substituting the value into the expression for $\angle NKL$. If you don't get the same measure as $\angle JKN$, you have made an error.

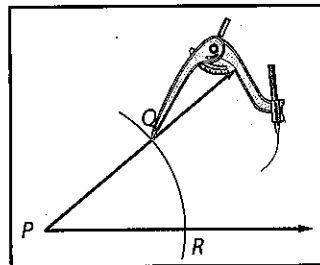
You can produce the angle bisector of any angle without knowing the measure of the angle.

Construction Bisect an Angle

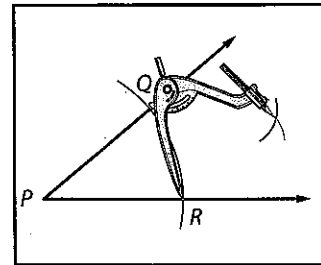
Step 1 Draw an angle on your paper. Label the vertex as P . Put your compass at point P and draw a large arc that intersects both sides of $\angle P$. Label the points of intersection Q and R .



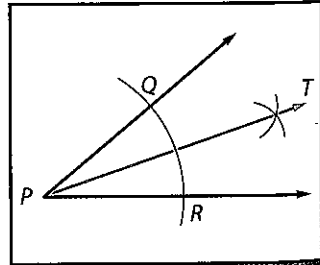
Step 2 With the compass at point Q , draw an arc in the interior of the angle.



Step 3 Keeping the same compass setting, place the compass at point R and draw an arc that intersects the arc drawn in Step 2. Label the point of intersection T .



Step 4 Draw \overrightarrow{PT} . \overrightarrow{PT} is the bisector of $\angle P$.

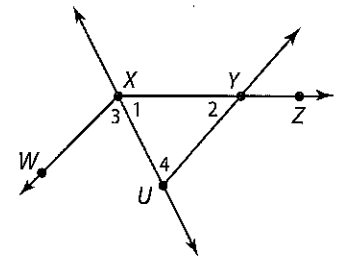


Check Your Understanding

= Step-by-Step Solutions begin on page R14.

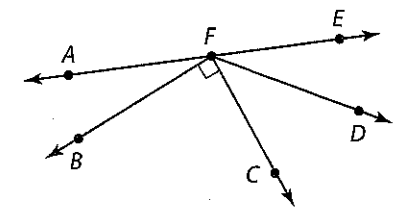
Example 1 Use the figure at the right.

1. Name the vertex of $\angle 4$.
2. Name the sides of $\angle 3$.
3. What is another name for $\angle 2$?
4. What is another name for $\angle UXY$?



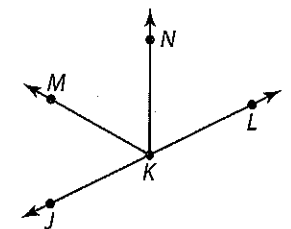
Example 2 Copy the diagram shown, and extend each ray. Classify each angle as *right*, *acute*, or *obtuse*. Then use a protractor to measure the angle to the nearest degree.

5. $\angle CFD$
6. $\angle AFD$
7. $\angle BFC$
8. $\angle AFB$

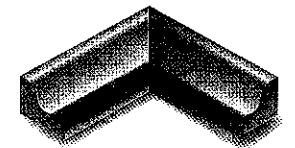


Example 3 **ALGEBRA** In the figure, \overrightarrow{KJ} and \overrightarrow{KL} are opposite rays. \overrightarrow{KN} bisects $\angle LKM$.

9. If $m\angle LKM = 7x - 5$ and $m\angle NKM = 3x + 9$, find $m\angle LKM$.
10. If $m\angle NKL = 7x - 9$ and $m\angle JKM = x + 3$, find $m\angle JKN$.



11. **CCSS PRECISION** A miter cut is used to build picture frames with corners that meet at right angles.
 - a. José miteres the ends of some wood for a picture frame at congruent angles. What is the degree measure of his cut? Explain and classify the angle.
 - b. What does the joint represent in relation to the angle formed by the two pieces?



Practice and Problem Solving

Extra Practice is on page R1.

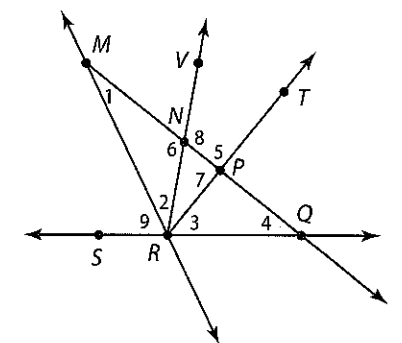
Example 1 For Exercises 12–29, use the figure at the right.

- Name the vertex of each angle.
12. $\angle 4$
 13. $\angle 7$
 14. $\angle 2$
 15. $\angle 1$

- Name the sides of each angle.
16. $\angle TPQ$
 17. $\angle VNM$
 18. $\angle 6$
 19. $\angle 3$

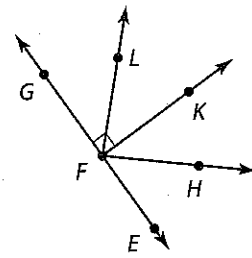
- Write another name for each angle.
20. $\angle 9$
 21. $\angle QPT$
 22. $\angle MQS$
 23. $\angle 5$

24. Name an angle with vertex N that appears obtuse.
25. Name an angle with vertex Q that appears acute.
26. Name a point in the interior of $\angle VRQ$.
27. Name a point in the exterior of $\angle MRT$.
28. Name a pair of angles that share exactly one point.
29. Name a pair of angles that share more than one point.



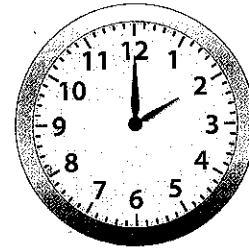
Example 2

Copy the diagram shown, and extend each ray. Classify each angle as *right*, *acute*, or *obtuse*. Then use a protractor to measure the angle to the nearest degree.



- 30. $\angle GFK$
- 31. $\angle EFK$
- 32. $\angle LFK$
- 33. $\angle EFH$
- 34. $\angle GFH$
- 35. $\angle EFL$

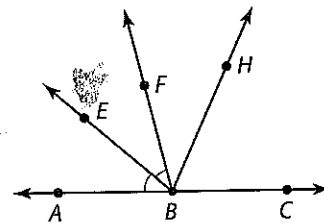
36. **CLOCKS** Determine at least three different times during the day when the hands on a clock form each of the following angles. Explain.



- a. right angle
- b. obtuse angle
- c. congruent acute angles

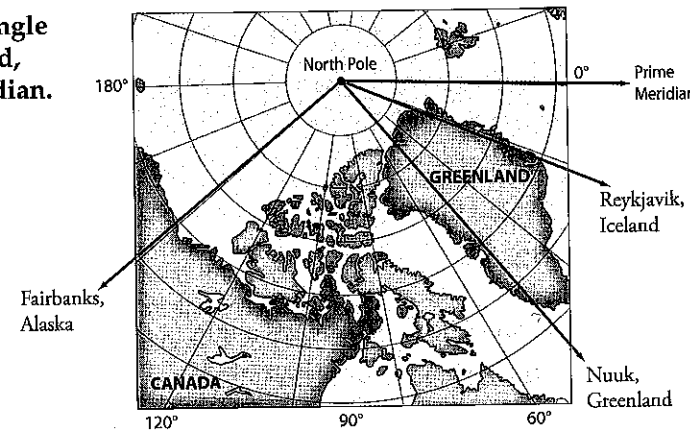
Example 3

ALGEBRA In the figure, \overrightarrow{BA} and \overrightarrow{BC} are opposite rays. \overrightarrow{BH} bisects $\angle EBC$.



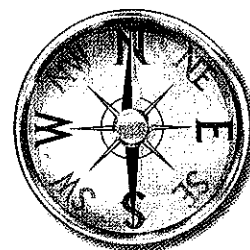
- 37. If $m\angle ABE = 2n + 7$ and $m\angle EBF = 4n - 13$, find $m\angle ABE$.
- 38. If $m\angle EBH = 6x + 12$ and $m\angle HBC = 8x - 10$, find $m\angle EBH$.
- 39. If $m\angle ABF = 7b - 24$ and $m\angle ABE = 2b$, find $m\angle EBF$.
- 40. If $m\angle EBC = 31a - 2$ and $m\angle EBH = 4a + 45$, find $m\angle HBC$.
- 41. If $m\angle ABF = 8s - 6$ and $m\angle ABE = 2(s + 11)$, find $m\angle EBF$.
- 42. If $m\angle EBC = 3r + 10$ and $m\angle ABE = 2r - 20$, find $m\angle EBF$.

43. **MAPS** Estimate the measure of the angle formed by each city or location listed, the North Pole, and the Prime Meridian.



- a. Nuuk, Greenland
- b. Fairbanks, Alaska
- c. Reykjavik, Iceland
- d. Prime Meridian

44. **TOOLS** A compass rose is a design on a map that shows directions. In addition to the directions of north, south, east, and west, a compass rose can have as many as 32 markings.

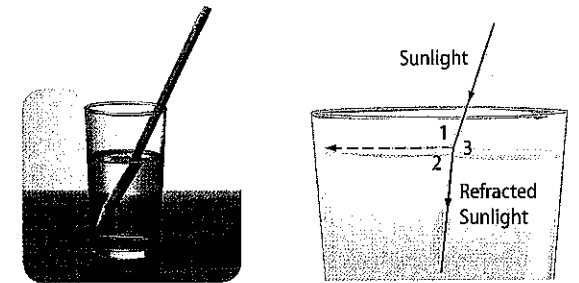


- a. With the center of the compass as its vertex, what is the measure of the angle between due west and due north?
- b. What is the measure of the angle between due north and north-west?
- c. How does the north-west ray relate to the angle in part a?

Plot the points in a coordinate plane and sketch $\triangle XYZ$. Then classify it as *right*, *acute*, or *obtuse*.

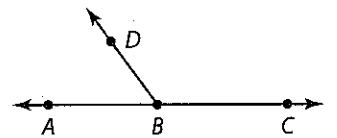
- 45. $X(5, -3), Y(4, -1), Z(6, -2)$
- 46. $X(6, 7), Y(2, 3), Z(4, 1)$

PHYSICS When you look at a pencil in water, it looks bent. This illusion is due to *refraction*, or the bending of light when it moves from one substance to the next.



- a. What is $m\angle 1$? Classify this angle as *acute*, *right*, or *obtuse*.
- b. What is $m\angle 2$? Classify this angle as *acute*, *right*, or *obtuse*.
- c. Without measuring, determine how many degrees the path of the light changes after it enters the water. Explain your reasoning.

48. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the relationship of angles that compose opposite rays.

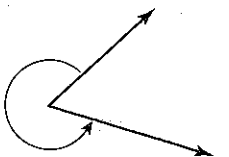


- a. **Geometric** Draw four lines, each with points A, B, and C. Draw \overrightarrow{BD} for each line, varying the placement of point D. Use a protractor to measure $\angle ABD$ and $\angle DBC$ for each figure.
- b. **Tabular** Organize the measures for each figure into a table. Include a row in your table to record the sum of these measures.
- c. **Verbal** Make a conjecture about the sum of the measures of the two angles. Explain your reasoning.
- d. **Algebraic** If x is the measure of $\angle ABD$ and y is the measure of $\angle DBC$, write an equation that relates the two angle measures.

H.O.T. Problems Use Higher-Order Thinking Skills

49. **OPEN ENDED** Draw an obtuse angle named $\angle ABC$. Measure $\angle ABC$. Construct an angle bisector \overrightarrow{BD} of $\angle ABC$. Explain the steps in your construction and justify each step. Classify the two angles formed by the angle bisector.

50. **CHALLENGE** Describe how you would use a protractor to measure the angle shown.



51. **ARGUMENTS** The sum of two acute angles is *sometimes*, *always*, or *never* an obtuse angle. Explain.

52. **CHALLENGE** \overrightarrow{MP} bisects $\angle LMN$, \overrightarrow{MQ} bisects $\angle LMP$, and \overrightarrow{MR} bisects $\angle QMP$. If $m\angle RMP = 21$, find $m\angle LMN$. Explain your reasoning.

53. **WRITING IN MATH** Rashid says that he can estimate the measure of an acute angle using a piece of paper to within six degrees of accuracy. Explain how this would be possible. Then use this method to estimate the measure of the angle shown.

