Name____________________________________

Honors Algebra 2 — Ch 8(pt 2) Notes Packet

Section 8.5: Solving Rational Equations and Inequalities

We have worked with rational expressions and functions in the following ways***:

- Simplifying
- Adding and subtracting
- Multiplying and dividing (including complex fractions)
- Transformations of f(x) = 1/x
- Graphing: including vertical/ horizontal asymptotes, holes, zeros…
- Direct, inverse and joint variation

*** can you do all these things? If not, REVIEW from last semester!

Now we will be SOLVING rational equations for the value(s) of x that makes the equations true. Knowing what you do about solving fractions (and since solving fractions is a similar process to solving rationals), briefly jot down your plan for solving in the margin before trying out your plan:

Ex1: Solve for x. Box final answers.

A. $\frac{3}{x+1} = \frac{9}{4x+5}$

B. $1 - \frac{8}{x-5} = \frac{3}{x}$

C. $\frac{5x}{x-2} = 7 + \frac{10}{x-2}$

D. $\frac{6}{x-3} = \frac{8x^2}{x^2-9} - \frac{4x}{x+3}$

As you may have noticed, sometimes solving rational equations requires extensive manipulation which creates ______________________________________, that are not solutions to the given problem. You must always check for these before providing your final answers. ✔

Ex2: Applications

A. The coolant in a car radiator is a mixture of pure antifreeze and water. The recommended mixture for your car is 50% antifreeze. If you have a mixture of 7 liters of coolant that is 40% antifreeze, how much pure antifreeze should you add to the mixture to bring it up to the recommended level?

B. Natalia can complete an inventory of a stock room in 8 hours. When Natalia and Antonio work together, they can finish in 4 ½ hours. How long would it take Antonio to complete the stock room inventory alone?

Solving Rational Inequalities:

- Rewrite the inequality so that one side = ________
- Write the other side as a single simplified ________________________________
- Identify all __________________________ (x-values where numerator and denominator = 0)
- Using a number line, _____________________ in each interval created by the critical values
- Write inequalities that express the ________________________________

Ex3: Solve the inequality algebraically

A. $\frac{6}{x-2} \geq -4$

B. $\frac{5}{x+3} \geq \frac{4}{x+2}$

Try This #1:

A. $\dfrac{x-3}{x+5} = \dfrac{x}{x+2}$

B. $\dfrac{6x}{x+4} + 4 = \dfrac{2x+2}{x-1}$

Try This #2:

An alloy is formed by mixing two or more metals. Sterling silver is an alloy of 92.5% pure silver and 7.5% copper. Jewelry silver is an alloy of 80% pure silver and 20% copper. How much pure silver should you mix with 15 ounces of jewelry silver to make sterling silver?

Try This #3: Solve the inequality

$\dfrac{9}{x+3} > 6$

Section 8.6: Radical Expressions and Rational Exponents

The n^{th} root of a real number *a* is written as: where n = & a =

Principal square root =

The number and type of real roots is determined by ___________________ & ____________________

INDEX	RADICAND	# of Real Roots

Rational Exponent: exponents written as a fraction $\frac{m}{n}$, where m & n integers and n ≠ 0

PROPERTIES of N^{th} ROOTS **Assume positive values**

Product Property of Roots: (n^{th} root of a product = product of n^{th} roots)	$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$	$\sqrt[3]{16x^5}$
Quotient Property of Roots: (n^{th} root of a quotient = quotient of n^{th} roots)	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$	$\sqrt[4]{\frac{81}{x^4}}$
n^{th} root → $\frac{1}{n}$ as exponent	$a^{\frac{1}{n}} = \sqrt[n]{a}$	$16^{\frac{1}{4}}$
n^{th} root raised to power of m → $\frac{m}{n}$ as exponent	$a^{\frac{m}{n}} = \sqrt[n]{a^m}$	$8^{\frac{2}{3}}$

Ex 1: Simplify. Assume positive values.

A. $\sqrt[4]{\frac{16x^8}{5}}$

B. $\sqrt[3]{7} \cdot \sqrt[3]{x^4}$

C. $(-\ 32)^{\frac{3}{5}}$

D. $\sqrt[8]{12^4}$

Review:

Product of Powers: if you are multiplying with the same → add exponents	$a^m \cdot a^n = a^{m+n}$
Quotient of Powers: if you are dividing with the same → subtract exponents	$\frac{a^m}{a^n} = a^{m-n}$
Power of Product/Quotient: product/quotient taken to a power → distribute exponent	$(a \cdot b)^m = a^m \cdot b^m$ OR $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
Power of Power: raising one power to another power → multiply exponents	$(a^m)^n = a^{m \cdot n}$

Ex 2: Simplify.

A. $7^{\frac{7}{9}} \cdot 7^{\frac{11}{9}}$

B. $(-8)^{-\frac{1}{3}}$

C. $\dfrac{16^{\frac{3}{4}}}{16^{\frac{5}{4}}}$

D. $\left(64^{\frac{1}{3}}\right)^{\frac{1}{2}}$

E. $\dfrac{\sqrt{9a^5}}{(128b^4)^{\frac{1}{2}}}$

F. $(\sqrt{5} - \sqrt{8})(\sqrt{5} + \sqrt{2})$

Ex 3: Applications

Radium-226 is a form of a radioactive element that decays over time. An initial sample in grams decays over t years to an ending amount using the function $A(x) = 500(2)^{-t/1600}$. How much Radium-266 is left after 800 years? Round to tenths.

Try This #4:

A. List all real 4th roots of 64 B. List all real cube roots of -216 C. List all real 4th roots of -1024

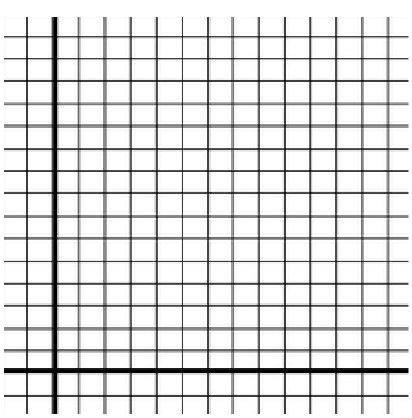
Try This #5: Simplify. Assume positive values.

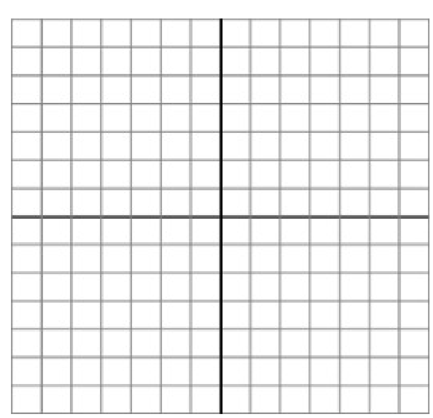
A. $\sqrt[4]{243x^{12}}$ B. $625^{\frac{3}{4}}$ C. $\dfrac{(12)^{\frac{1}{4}}}{\sqrt[4]{9a^3}}$

Section 8.7: Radical Functions

A **RADICAL FUNCTION** has a radical expression in its function rule.
Each nth root has a separate parent function. So, a square root parent function is $f(x) = \sqrt{x}$ and a cube root parent function is $f(x) = \sqrt[3]{x}$.

Radical functions are initially formed by finding the inverse of a polynomial.





Transformations:

Transformations of the Square-Root Parent Function $f(x) = \sqrt{x}$		
Transformation	**$f(x)$ Notation**	**Examples**
Vertical translation	$f(x) + k$	$y = \sqrt{x} + 3$ 3 units up $y = \sqrt{x} - 4$ 4 units down
Horizontal translation	$f(x - h)$	$y = \sqrt{x - 2}$ 2 units right $y = \sqrt{x + 1}$ 1 unit left
Vertical stretch/compression	$af(x)$	$y = 6\sqrt{x}$ vertical stretch by 6 $y = \frac{1}{2}\sqrt{x}$ vertical compression by $\frac{1}{2}$
Horizontal stretch/ compression	$f\left(\frac{1}{b}x\right)$	$y = \sqrt{\frac{1}{5}x}$ horizontal stretch by 5 $y = \sqrt{3x}$ horizontal compression by $\frac{1}{3}$
Reflection	$-f(x)$ $f(-x)$	$y = -\sqrt{x}$ across x-axis $y = \sqrt{-x}$ across y-axis

Summary:

$f(x) = \sqrt{\quad}$

Ex 1: Describe the transformations from $f(x) = \sqrt{x}$. State the domain and range.

A. $g(x) = \sqrt{x + 3}$

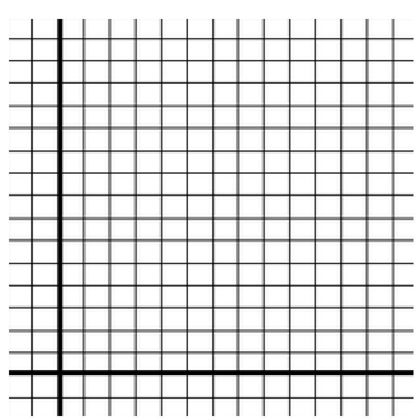
B. $g(x) = 2\sqrt{-x} + 1$

Ex 2: Write the daughter function if the parent function is a cubic function that has been stretched horizontally by a factor of 5, reflected over the x axis and translated 4 units down.

GRAPHING RADICAL INEQUALITIES:

- Graph boundary curve with appropriate solid/dashed line
- Test points to determine shading

Ex 3: Graph $y > 2\sqrt{x - 3}$

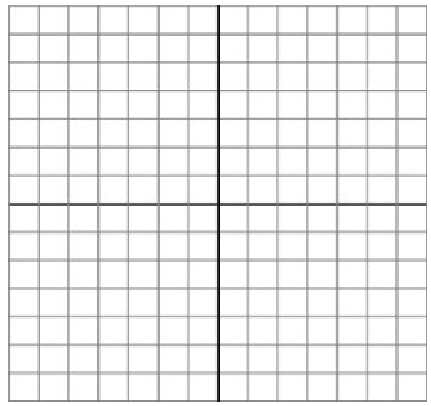


Try This #6: Describe the transformation from a square root parent function. State the domain and range.

A. $g(x) = \sqrt{x} + 5$

B. $g(x) = -\sqrt{x-4}$

Try This #7: Graph $y = \sqrt[3]{x} - 1$



Section 8.8: Solving Radical Equations and Inequalities

A **RADICAL EQUATION** is an equation that has a variable located within the radicand to solve for.

To solve a radical equation (or an equation with rational exponents) some of the following solving strategies may be used:

- Isolate the radical, if possible
- Raise both sides of the equation to the power of the index of the radical or raise both sides of the equation to the reciprocal of the rational exponent [Recognize that extraneous solutions may be introduced]
- Convert between rational exponents and radicals
- Simplify by expanding binomials and trinomials using Pascal's triangle or box method
- CHECK YOUR SOLUTIONS!!!

Ex 1: Solve.

A. $5 + \sqrt{x+1} = 16$

B. $\sqrt{-3x+33} = 5 - x$

C. $\sqrt{7x+2} = 3\sqrt{3x-2}$

D. $2x = (4x+8)^{\frac{1}{2}}$

Solutions of RADICAL INEQUALITIES can be found algebraically or graphically.

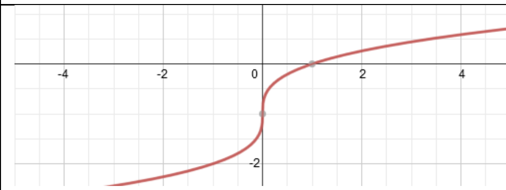
Ex 2: Solve $\sqrt{x-3} + 2 \leq 5$.

Try This # 8: Solve.

A. $7\sqrt[3]{5x-7} = 84$

B. $(5x+7)^{\frac{1}{3}} = 3$

Try This Answers:

1A: x = -1	5A: $3x^4 \cdot \sqrt[4]{3}$	#7:
1B: x = -3/2 & 2	5B: 125	
#2: 25 oz	5C: $\frac{\sqrt[4]{2a}}{a}$	
#3: -3 < x < -3/2	6A: vertical shift up 5, D: [0, ∞), R: [5, ∞)	
4A: ±2√2	6B: reflection over x-axis	
4B: -6	horizontal shift right 4	8A: x = 347
4C: no real solutions	D: [4, ∞), R: (∞, 0]	8B: x = 4