Name____________________________________

Honors Algebra 2 Ch 10 Notes Packet

Section 10.1: Introduction to Conic Sections

Use the table on p. 724 to answer:
What is the midpoint formula?

What is the distance formula?

Examples:

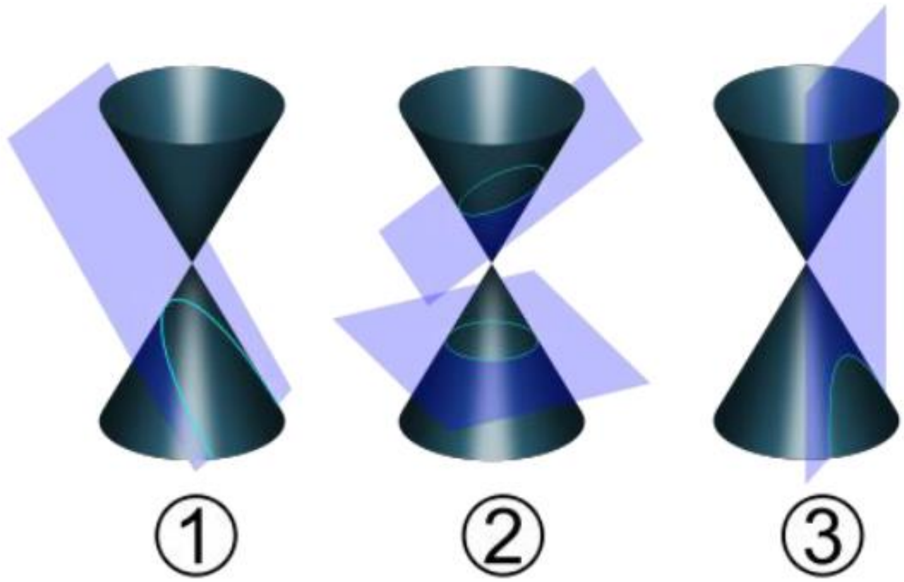
A. Find the center and radius of a circle that has a diameter with endpoints of (2, 6) and (14, 22).

B.

C.

Section 10.2: Circles

Conic Sections: a **conic section** (or just **conic**) is a curve obtained by intersecting a cone with a plane. The conic sections were named and studied as long ago as 200 BC, when Apollonius of Perga undertook a systematic study of their properties. All the variations in the shape of a conic section can be obtained by varying the slope of the plane intersecting the conical surface.



Types of conic sections:
1. Parabola
2. Circle and ellipse
3. Hyperbola

Circle Vocab:
LOCUS: a set of points that satisfy a given set of conditions **CIRCLE:** locus of points in a plane at a given distance from a fixed point called the **CENTER** **RADIUS:** distance from the **CENTER** to any point on the circle **CONCENTRIC CIRCLES:** circles that have the same center but not the same radius **TANGENT:** a line in the same plane of a circle that intersects the circle at exactly one point. The tangent to a circle is perpendicular to the radius at the **POINT OF TANGENCY**

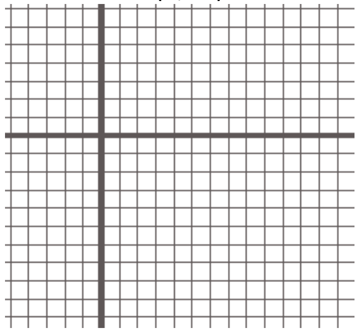
STANDARD FORM of the EQUATION of a CIRCLE: where (h,k) is the center & r = radius

RECOGNIZING THE EQUATION OF A CIRCLE:

***** Helpful Formulas to Recall/ Review: Distance Formula and Midpoint Formula**

Ex1: Write the equation of the circle in standard form:

A. With center (4, -1) and radius 6 and then graph



B. With center (- 4, 11) and containing (5, -1)

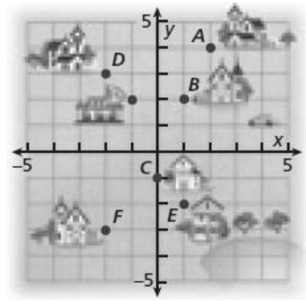
C. With diameter that has endpoints of (- 1, 1) and (5, 13)

Ex2: Interpret the difference in:

A. $(x - h)^2 + (y - k)^2 = r^2$ vs $(x - h)^2 + (y - k)^2 \geq r^2$ vs $(x - h)^2 + (y - k)^2 > r^2$

B. $(x - h)^2 + (y - k)^2 < r^2$ vs $(x - h)^2 + (y - k)^2 > r^2$

Ex3: **A.** Raul and his friends are having a pizza party and will decide where to have the party based on the delivery area of the pizza restaurant. Suppose that the pizza restaurant is located at the point $(-1, 2)$ and the letters represent the homes of Raul and his friends. Use the equation of a circle to find the houses that are within a 3-mile radius and will get free delivery.



B. Use the map from Ex 3 A to determine which homes are within four miles of a restaurant located at (- 1, 1).

Ex4: Write the y-intercept equation of the line tangent to circle $(x - 1)^2 + (y + 3)^2 = 13$ at (4, -5)

Section 10.3: Ellipses

Ellipse Vocab:

ELLIPSE: locus of all points in a plane such that the sum of the distances from two fixed points (foci) is constant

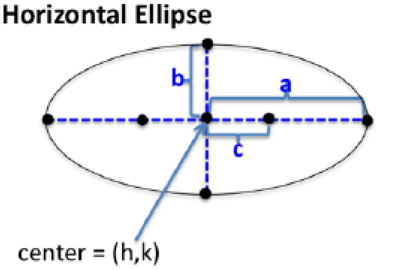
An ellipse has two **AXES OF SYMMETRY**, the **MAJOR AXIS** & **MINOR AXIS**. The point where the two axes intersect is the **CENTER** of the ellipse and the center divides the major & minor axes into two congruent segments

- the major axis is the longest axis and it contains the **FOCI**. Its length is 2a and a is the distance from the center to an end of the major axis
    - the endpoints of the major axis are called vertices
- the minor axis is the shortest axis and its length is 2b. b is the distance from the center to an end of the minor axis
    - the endpoints of the minor axis are called co-vertices
- the foci (plural form of **FOCUS)** are the two fixed points and can be found using the formula $c^2 = a^2 - b^2$ where c is the distance from the center to a focus point

RECOGNIZING THE EQUATION OF AN ELLIPSE:

STANDARD FORM of the EQUATION of an ELLIPSE:

Horizontal Ellipse

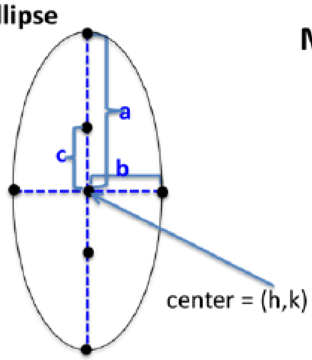


Major Axis Parallel to x-axis

- Vertices (h±a, k)
- Co-vertices (h, k±b)
- Foci (h±c, k)

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

Vertical Ellipse



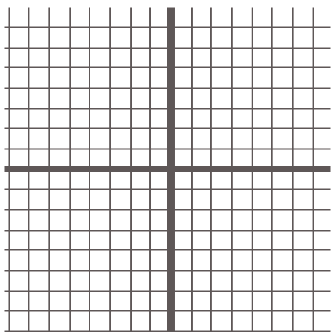
Major Axis Parallel to y-axis

- Vertices (h, k±a)
- Co-vertices (h±b, k)
- Foci (h, k±c)

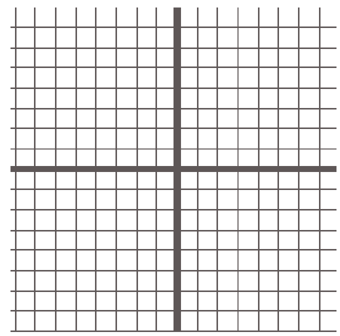
$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

Ex1: Find the constant sum for an ellipse with foci $F_1 (3, 0)$ and $F_2 = (24, 0)$ and a point on the ellipse P(9, 8).

Ex2 A. Graph $\frac{x^2}{16} + \frac{(y+4)^2}{9} = 1$



B. Graph $\frac{(x-3)^2}{16} + \frac{(y-2)^2}{25} = 1$



Ex3: Write the equation for each ellipse described.

A. Center at origin, vertex (6, 0) & co-vertex (0, 4)

B. Center at origin, focus (0, 3) & co-vertex (5, 0)

C. center is (-5, 1) its major axis is 10 units long and parallel to the x-axis and its minor axis is 6 units long

D. Vertices (3, 6)&(3,-2) Foci (3, 5)&(3,-1)

Section 10.4: Hyperbolas

HYPERBOLA VOCAB

HYPERBOLA: locus of all points in a plane such that the absolute value of the differences of the distance from two fixed points (foci) is constant → $d = | PF_1 - PF_2 |$

- a hyperbola has two **AXES OF SYMMETRY**, the **TRANSVERSE AXIS & CONJUGATE AXIS.** The point where the two axes intersect is the **CENTER** of the hyperbola – the center is also the midpoint of the segment whose endpoints are the foci
- the foci are the two fixed points and can be found using $\mathbf{c^2 = a^2 + b^2}$ where c is the distance from the center to a focus point
- the endpoints of the transverse axis are called **VERTICES.** The transverse axis contains the vertices (and if extended, the foci also) and the length of the transverse axis is 2a
- the conjugate axis is perpendicular to the transverse axis at the center and separates the hyperbola into 2 **BRANCHES**. The endpoints of the conjugate axis are called **CO-VERTICES** and its length is 2b

ASYMPTOTE: an imaginary line that a graph approaches but never reaches (as inputs get larger and larger or smaller and smaller)

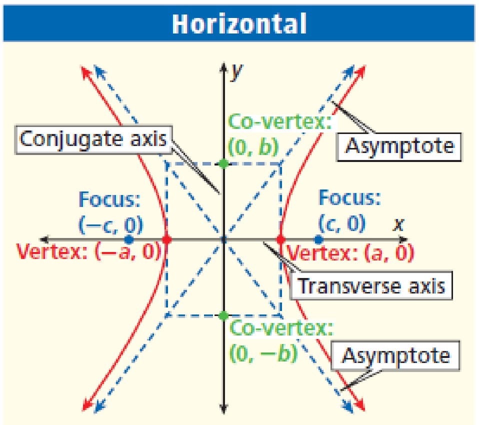
Standard Form for the Equation of a Hyperbola Center at (h, k)

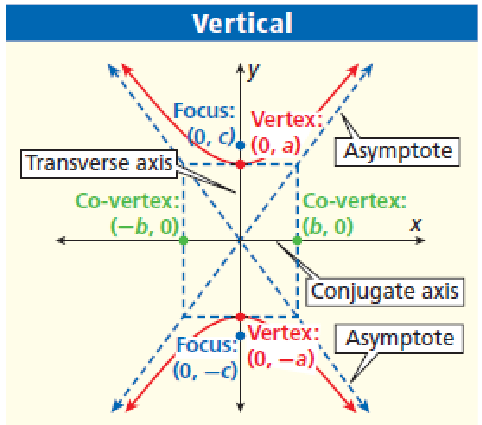
TRANSVERSE AXIS	HORIZONTAL	VERTICAL
Equation	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
Vertices	$(h+a, k), (h-a, k)$	$(h, k+a), (h, k-a)$
Foci	$(h+c, k), (h-c, k)$	$(h, k+c), (h, k-c)$
Co-vertices	$(h, k+b), (h, k-b)$	$(h+b, k), (h-b, k)$
Asymptotes	$y-k = \pm\frac{b}{a}(x-h)$	$y-k = \pm\frac{a}{b}(x-h)$

As the parameters change the hyperbola is transformed:

Parameter	Transformation
h	Translates the graph left for $h > 0$ and right for $h < 0$
k	Translates the graph up for $k > 0$ and down for $k < 0$
a	Stretches the graph in the direction of the transverse axis; as a increases, the vertices move farther apart.
b	Stretches the graph in the direction of the conjugate axis; as b increases, the co-vertices move farther apart.

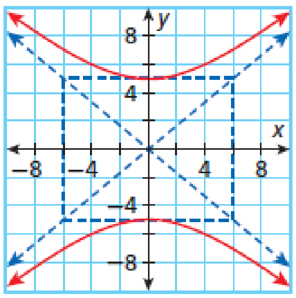
RECOGNIZING THE EQUATION OF A HYPERBOLA:



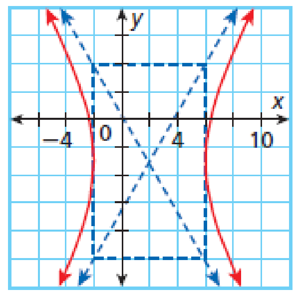


Ex1: Write the equation of the hyperbola shown/ described. Graph C & D.

A.

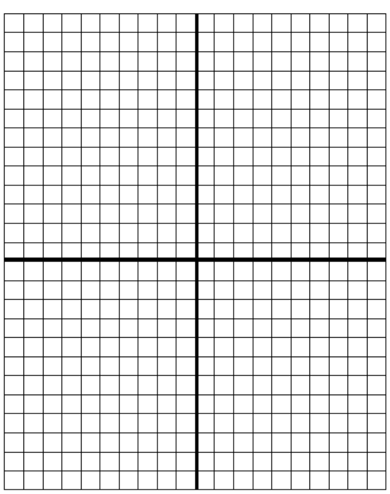


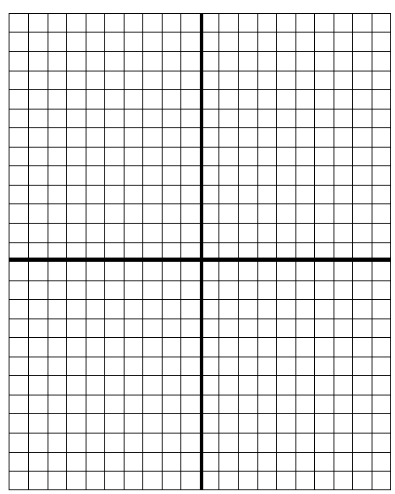
B.



C. Center is (-2,3), has a horizontal transverse axis of 12 units long and a conjugate axis of 20 units long

D. Vertices are (-4,2) and (-4,8) and whose conjugate axis is 10 units long.





Section 10.5: Parabolas

PARABOLA VOCAB

PARABOLA: locus of all points in a plane that are the same distance from a given point called the **FOCUS** to a given line called the **DIRECTRIX**

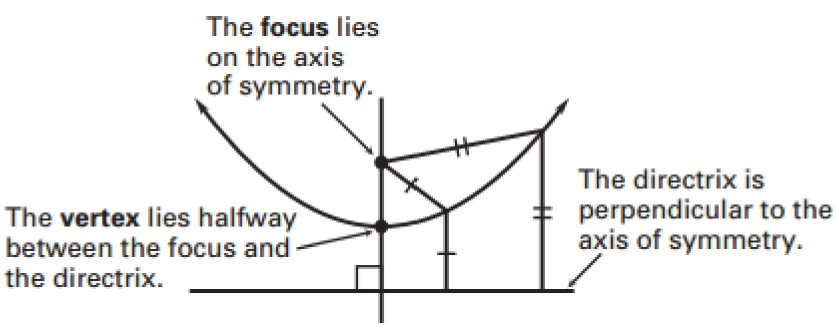
- p is the distance from the focus to the vertex and the distance from the vertex to the directrix

In the equation only one of the variables is squared

- if the parabola **opens up or down**, x is squared
- if the parabola **opens right or left**, y is squared

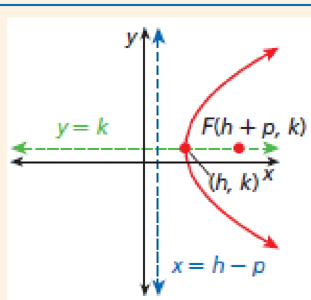
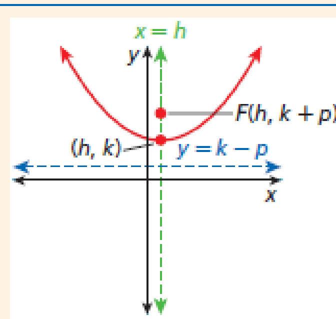
AXIS OF SYMMETRY: the line that passes through the vertex of the parabola and divides the parabola into two matching halves

- axis of symmetry is x = h if the parabola opens up or down
- axis of symmetry is y = k if the parabola opens right or left



RECOGNIZING EQUATION OF A PARABOLA:

Standard Form for the Equation of a Parabola — Vertex at (h, k)

AXIS OF SYMMETRY	HORIZONTAL $y = k$	VERTICAL $x = h$
Equation		
Direction	Opens right if $p > 0$ Opens left if $p < 0$	Opens upward if $p > 0$ Opens downward if $p < 0$
Focus	$(h + p, k)$	$(h, k + p)$
Directrix	$x = h - p$	$y = k - p$
Graph		

Ex1: Find the coordinates of the vertex, value of p and state the direction of the opening for:

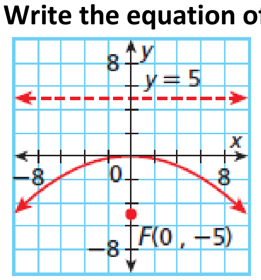
A. $(x + 2) = \frac{1}{2}(y + 5)^2$

B. $(x - 3)^2 = -8(y - 4)$

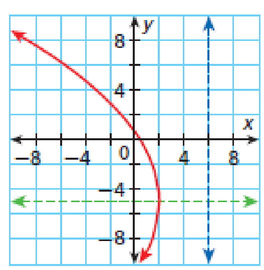
Ex2: Using the distance formula, write the equation of the parabola with a focus F(2, 4) and directrix y = - 4.

Ex3: Write the equation of the parabola shown/ described.

A.



B.

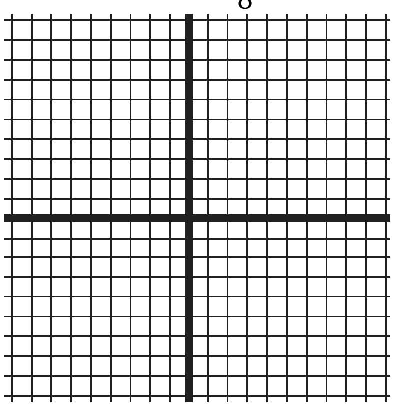


C. Focus (2,5) and directrix x = 4

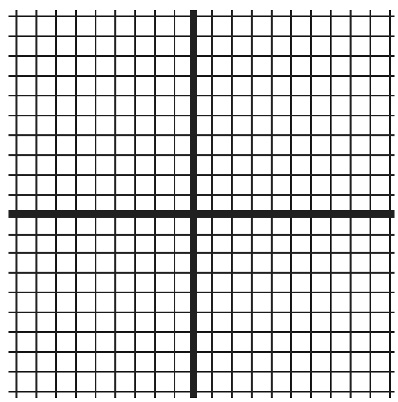
D. Vertex (4, 2) and focus (4, 3)

Ex4: Graph the parabola by finding the vertex, focus and equation of the directrix and AOS.

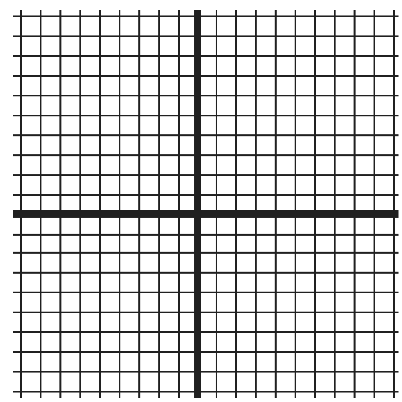
A. $x - 1 = \frac{1}{8}(y - 2)^2$



B. $(y + 3)^2 = -16(x - 5)$



C. $\mathrm{x}^2 - 8\mathrm{y} = 0$



Section 10.6: Identifying Conic Sections

Identifying Conics in Standard Form	
Circle:	Ellipse:
Hyperbola:	Parabola:

The **GENERAL FORM** of a conic section is $\mathbf{Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0}$
(where A, B, and C are not ALL equal to zero)

CONIC SECTION	COEFFICIENTS
Circle	$B^2 - 4AC < 0$, $B = 0$, and $A = C$
Ellipse	$B^2 - 4AC < 0$ and either $B \neq 0$ or $A \neq C$
Hyperbola	$B^2 - 4AC > 0$
Parabola	$B^2 - 4AC = 0$

Ex1: Identify the conic section

A. $\dfrac{(y-5)^2}{36} + \dfrac{(x+2)^2}{16} = 1$

B. $16(x-1)^2 = 144 + 9(y-2)^2$

C. $\dfrac{(x-3)^2}{8} + \dfrac{(y-2)^2}{8} = \dfrac{16}{50}$

D. $x + 4 = \dfrac{(y-2)^2}{10}$

E. $\dfrac{(x-6)^2}{36} = \dfrac{(y+4)^2 + 16}{16}$

F. $4x^2 - 10xy + 5y^2 + 12x + 20y = 0$

G. $12x^2 + 18y^2 + 24x - 30y - 50 = 0$

H. $9x^2 - 12xy + 4y^2 + 6x - 8y = 0$

- ✓ **General form is not easily graphed, so it is important to develop some skills to find the standard form of a conic section from the general form. We will begin to develop some of these skills now.**

Ex2: Write the equation of the conic section in standard form.

A. $6y^2 - 24y = 9 - 12x^2 - 36$

B. $9x^2 - 16y^2 - 90x - 64y + 17 = 0$

C. $4x^2 + 4y^2 - 24x + 16y = -51$

D. $y^2 + 16x + 4y - 44 = 0$

Section 10.7: Solving Nonlinear Systems

A system of nonlinear equations is two or more equations (at least one of which is not a linear equation) that are being solved simultaneously.
***Note that in a nonlinear system, one or more of your equations can be linear, just not ALL of them.

- We will primarily use the **substitution method** to solve a non-linear system. However, sometimes the **elimination method** is a viable option as well.

- Recall that the solution to a non-linear system is **all the points of intersection** of the graphs of the equations. Therefore, since we now have more than just lines, we can have a variety of numbers of solutions. The graph can intersect once, twice, several times or not at all. To verify the solution(s) to a system, look at the graph.

Examples of nonlinear systems:

Ex1: Solve the nonlinear system

A. $x^2 + y^2 = 100$
$y - x = 2$

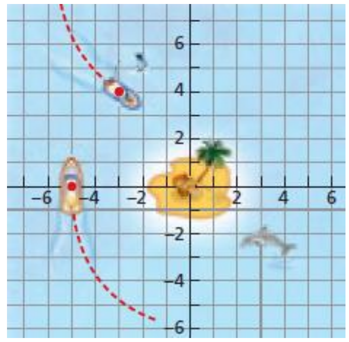
B. $x^2 + y^2 = 25$
$4x^2 + 9y^2 = 145$

C. $x^2 + y^2 = 100$
$y + 26 = \frac{1}{2}x^2$

D. $x^2 + 2y^2 = 12$
$xy = 4$

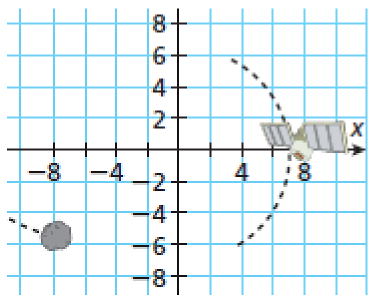
E.

A tour boat travels around a small island in a pattern that can be modeled by the equation $36x^2 + 25y^2 = 900$, with the island at the origin. Suppose that a fishing boat approaches the island on a path that can be modeled by the equation $y - 3 = \frac{1}{5}x^2$. Is there any danger of collision?



F.

Astronomy An asteroid is traveling toward Earth on a path that can be modeled by the equation $y = \frac{1}{28}x^2 - 7$. It approaches a satellite in orbit on a path that can be modeled by the equation $\frac{x^2}{49} + \frac{y^2}{51} = 1$. What are the coordinates of the points where the satellite and asteroid might collide?



Multi-Step The lake at a resort has an island near the center. A tour boat's path on the lake can be modeled by the equation $16x^2 + 9y^2 = 36$, with the island at the origin. If a canoe's path on the lake can be modeled by the equation $8x + 5y^2 = 20$, find the coordinates of the points on the lake where the boats might meet.

Try These: Solve the systems.

1. $\begin{cases} y = -4x \\ x+1 = \frac{1}{8}y^2 \end{cases}$

2. $\begin{cases} 4y = 5x \\ \frac{y^2}{9} - \frac{x^2}{9} = 1 \end{cases}$

3. $\begin{cases} 21x - 14y = 0 \\ \frac{3x^2}{16} + \frac{y^2}{36} = 1 \end{cases}$

4. $\begin{cases} x^2 + y^2 = 346 \\ x - 2 = \frac{1}{25}y^2 \end{cases}$

5. $\begin{cases} x^2 - y^2 = 40 \\ y + 10 = \frac{1}{7}x^2 \end{cases}$

6. $\begin{cases} y^2 + x^2 = 119 \\ x + 19 = \frac{1}{6}y^2 \end{cases}$

7. $\begin{cases} x - y = 6 \\ x^2 + y^2 = 132 \end{cases}$

8. $\begin{cases} y^2 = x^2 - 9 \\ x^2 + y^2 = 41 \end{cases}$

9. $\begin{cases} (x-3)^2 + y^2 = 17 \\ y + 7 = \frac{1}{2}(x-3)^2 \end{cases}$

10. $\begin{cases} 7x^2 - y^2 = -36 \\ x^2 - y^2 = -60 \end{cases}$

11. $\begin{cases} 3x^2 + 4y^2 = 1327 \\ x^2 + 2y^2 = 443 \end{cases}$

12. $\begin{cases} 8x^2 + 7y^2 = 2143 \\ x^2 + 5y^2 = -20 \end{cases}$

13.

Jordan is jogging on a path modeled by the equation $x^2 + y^2 = 2500$.

Katherine is jogging on a path modeled by the equation $\frac{y^2}{60^2} + \frac{x^2}{40^2} = 1$.

At what points do their paths intersect?