Name____________________________________

Honors Algebra 2 — Ch 12 Notes Packet: Sequences and Series

Section 12.1: Introduction to Sequences

Learning Target: We are learning about sequences.

Success Criteria:

- **I can find the *n*th term of a sequence.**
- **I can write rules for sequences.**

What are the next 3 terms in the pattern? How did you determine your answer?

2, 4, 6, 8, … 2, 4, 8, 16, … 3, 7, 11, 15, …

A SEQUENCE is…

We will be studying 2 types of sequences-

- Infinite Sequences:
- Finite Sequences:

A sequence can also be generated in 2 ways:

- Using an EXPLICIT formula
- Using a RECURSIVE formula

A sequence is another way to think of a function whose –

DOMAIN is

and RANGE is

Examples:

1. Tell if the given formula is recursive or explicit. Then find the first five terms of each sequence.

A. $a_1 = -2$
$a_n = 3a_{n-1} + 2$

B. $a_n = 3^n - 1$

C. $a_n = 3n + 5$

D. $a_1 = 2$
$a_n = -3a_{n-1}$

2. Write a recursive rule for the n^{th} term of each sequence. Then write an explicit rule for the n^{th} term of each sequence.

A. 1.5, 4, 6.5, 9, 11.5…

B. 5, 10, 20, 40, 80…

C. 7, 5, 3, 1, – 1…

D. $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5} \ldots$

Application:

1. To begin the half-time performance, a high school band marches onto the football field in a pyramid formation. The drum major leads the band alone in the first row. There are two band members in the second row, three in the third row, four in the fourth row, and so on.

A. Write an explicit formula for this situation.

B. If the pyramid formation has 10 rows. How many members does the band have?

2. A diagonal is an edge that joins two nonadjacent vertices in a polygon. The number of diagonals in the first four polygons is shown. Following the pattern, determine how many diagonals could be drawn in a polygon having twelve sides (dodecagon).

3. Your best friend tells you a rumor. Every hour a person who was told the rumor tells someone new. After a 6 hour period, how many people will have heard the rumor?

Section 12.2: Series and Summation Notation
Learning Targets: We are learning about series and summation notation. **Success Criteria:** • **I can evaluate the sum of a series expressed in sigma notation.**

A **SERIES** is the indicated sum of a sequence.

If a sequence is infinite, the numbers added may also be infinite and thus may not have defined sums. Often we find **PARTIAL SUMS** instead. A partial sum, S_n, is the sum of a specified number of terms of a sequence.

Let's think about the set of even numbers.

Notation for a series and its sum can be notated in 2 different ways:

S_1

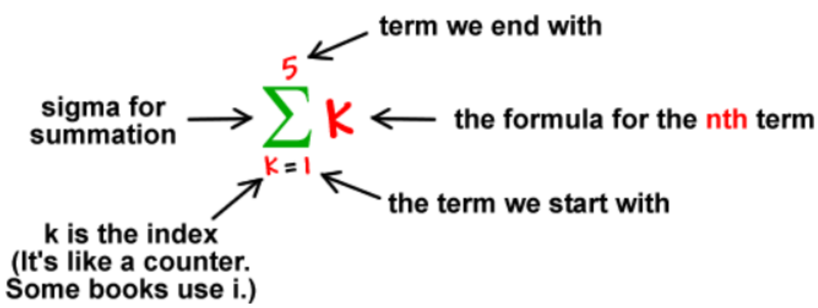
S_2

S_3

S_4

OR

Summation Notation which uses the Greek letter Σ (capital signma)



Caution!

For sequences with alternating signs:

Use $(-1)^{k+1}$ if $a_1 = +$.

Use $(-1)^{k}$ if $a_1 = -$.

Ex 1: Write each series in summation notation

A. $4 + 8 + 12 + 16 + 20$

B. $-1 + \frac{1}{4} - \frac{1}{9} + \frac{1}{16} - \frac{1}{25} + \frac{1}{36}$

Ex 2: Expand each series and evaluate the sums.

A. $\sum_{k=1}^{4}(2k-1)$

B. $\sum_{k=1}^{5}-5(2)^{k-1}$

<u>**You Try:**</u>

1.

2.

Some series are more common than others. Sum formulas for the following series have been derived:

Constant Series **Linear Series** **Quadratic Series**

Ex 3: Evaluate each series.

A. $\sum_{k=4}^{10} 6$

B. $\sum_{k=1}^{8} k$

C. $\sum_{k=1}^{12} k^2$

D. $\sum_{k=1}^{4} 2k^2 + k + 2$

Ex4: Sam is laying out patio stones in a triangular pattern. The first row has 2 stones and each row has 2 additional stones – one on each end. How many complete rows can he make with 144 stones?

Section 12.3: Arithmetic Sequences and Series
Learning Target: We are learning about arithmetic sequences and series. **Success Criteria:** • **I can find the indicated terms of an arithmetic sequence.** • **I can find the sums of arithmetic series.**

A sequence is an **ARITHMETIC SEQUENCE** if you can find a **common difference** (meaning each term changes by the same number). We can also see how that relationship connects to the linear function relationship.

Determine if the sequence is arithmetic. If so, write common difference and next term.

A. $-10, -4, 2, 8, 14, \ldots$ **B.** $-2, -5, -11, -20, -32, \ldots$ **C.** $1.9, 1.2, 0.5, -0.2, -0.9, \ldots$

General Rule for Arithmetic Sequences

The *n*th term a_n of an arithmetic sequence is given by

$$a_n = a_1 + (n - 1)d$$

where a_1 is the first term and d is the common difference.

This rule can be used to:

A. **Find the nth term given an arithmetic sequence of numbers**
B. **Find missing terms of a sequence**
C. **Find the nth term of a sequence given 2 terms**

❖ **Sample of A: Finding the nth term.**

Find the 10^{th} term of the arithmetic sequence 36, 29, 22, 15, 8…

Step 1: Find common difference

Step 2: Use formula to evaluate for requested term.

- **Sample of B: Finding missing terms.**

  Find the missing terms of the arithmetic sequence 11, ___, ___, ___, – 17

  Step 1: Find common difference

  Step 2: Find missing terms using d and a_1.

- **Sample of C: Find the nth term of a sequence given 2 terms.**

  Find the 6^{th} term of the arithmetic sequence when $a_9 = 120$ and $a_{14} = 195$.

  Step 1: Find the common difference.

  Step 2: Find a_1.

  Step 3: Write a rule for the sequence and evaluate to find the requested term.

Try these:

1. Find the 12th term in the arithmetic sequence.

A. $20, 14, 8, 2, -4, \ldots$

B. $-3, -5, -7, -9, \ldots$

2. Find the missing terms for the arithmetic sequence 17, ____, ____, ____, – 7

3. Find the 5th term when $a_8 = 85$ and $a_{14} = 157.$

Let's revisit how a sequence is related to a series… If you have the terms of an arithmetic sequences you can create an ARITHMETIC SERIES by finding the sum of the terms of the indicated sequence.

Sum of the First n Terms of an Arithmetic Series

WORDS	NUMBERS	ALGEBRA
The sum of the first n terms of an arithmetic series is the product of the number of terms and the average of the first and last terms.	The sum of $2 + 4 + 6 + 8 + 10$ is $5\left(\frac{2+10}{2}\right) = 5(6) = 30.$	$S_n = n\left(\frac{a_1 + a_n}{2}\right),$ where n is the number of terms, a_1 is the first term, and a_n is the nth term.

To find the sum of an arithmetic series using either notation, you can use the formula above.

A. For S_n notation:
Find S_{12} = 31, 20, 9, – 2, – 13…

B. For Σ notation:

$$\sum_{k=1}^{10} 3k + 4$$

Try these:

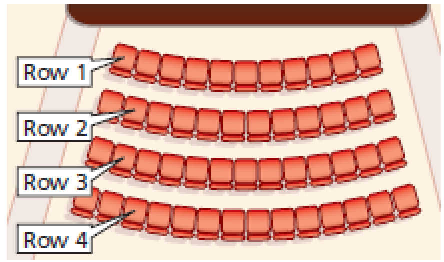
A. S_{18} for $13 + 2 + (-9) + (-20) + \dots$

B. $\sum_{k=1}^{15} (5 + 2k)$

Application:

Theater Application

The number of seats in the first 14 rows of the center orchestra aisle of the Marquis Theater on Broadway in New York City form an arithmetic sequence as shown.



A How many seats are in the 14th row?

B How many seats in total are in the first 14 rows?

Section 12.4: Geometric Sequences and Series
Learning Target: We are learning about geometric sequences and series. Success Criteria: • I can find terms of a geometric sequence, including geometric means. • I can find the sums of geometric series.

A **GEOMETRIC SEQUENCE** is a sequence whose successive terms have a **constant common ratio** (where $r \neq 1$).

A common ratio is found by dividing any term by the <u>previous</u> term.

Geometric sequences are <u>exponential functions</u> with sequential natural numbers as the **domain**.

Ex1: Determine if the sequence is arithmetic, geometric or neither. Find the common ratio or difference when possible.

A. 100, 93, 86, 79…

B. 180, 90, 60, 15…

C. 5, 1, 0.2, 0.04…

<u>RECURSIVE</u> **vs** **<u>EXPLICIT</u>**

Ex 2: Find the 9th term in each sequence.

A. 3, 12, 48, 192…

B. $\frac{3}{4}, \frac{-3}{8}, \frac{3}{16}, \frac{-3}{32}, \frac{3}{64}$

Find the 6th term in the sequence.

A. $a_2 = 4$ and $a_5 = 108$

B. $a_4 = 162$ and $a_8 = 13{,}122$

C. $a_3 = 36$ and $a_5 = 324$

If **a** and **b** are positive terms of a geometric sequence with exactly 1 term between them then the **GEOMETRIC MEAN** = $\sqrt{a \bullet b}$

**** geometric mean is NOT an arithmetic mean or average…

Ex 4: Find the geometric mean of the 2 numbers

A. 16 and 25

B. $\frac{4}{9}$ and $\frac{25}{36}$

C. 6 and 18

The sum of a **GEOMETRIC SERIES** can be found using: $S_n = a_1\left(\frac{1-r^n}{1-r}\right)$

Ex 5: Evaluate the geometric series.

A. S_8 for 1 + 2 + 4 + 8 …

B. $$\sum_{k=1}^{6}\left(\frac{1}{2}\right)^{k-1}$$

C. $$\sum_{k=1}^{6} -3(2)^{k-1}$$

D. S_6 for $2+1+\frac{1}{2}+\frac{1}{4}\ldots$

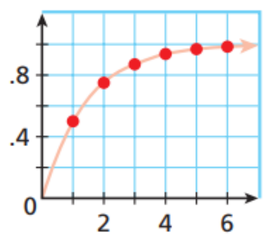
Ex 6: A math teacher earned $32,000 in his first year of teaching and earned a 1.5% raise each successive year. How much did he earn in his 20th year? What were his total earnings over his 20 year career?

Section 12.5: Infinite Geometric Series
Learning Target: We are learning about infinite geometric series. **Success Criteria:** • **I can find sums of infinite geometric series.**

An infinite geometric has infinitely many terms.

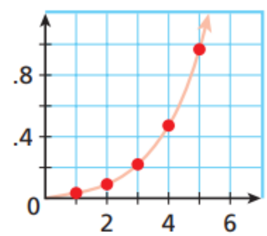
$$S_n = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots$$

Partial Sums						
n	1	2	3	4	5	6
S_n	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{15}{16}$	$\frac{31}{32}$	$\frac{63}{64}$



$$R_n = \frac{1}{32} + \frac{1}{16} + \frac{1}{8} + \frac{1}{4} + \frac{1}{2} + \cdots$$

Partial Sums						
n	1	2	3	4	5	6
R_n	$\frac{1}{32}$	$\frac{3}{32}$	$\frac{7}{32}$	$\frac{15}{32}$	$\frac{31}{32}$	$\frac{63}{32}$



Some infinite geometric series CONVERGE to a LIMIT and other DIVERGE.

Ex 1: Determine whether each geometric series converges or diverges.

A. $\frac{81}{625} + \frac{27}{125} + \frac{9}{25} + \frac{3}{5} + 1 + \cdots$

B. $1 - \frac{3}{5} + \frac{9}{25} - \frac{27}{125} + \frac{81}{625} - \cdots$

C. $256 + 64 + 16 + 4 + 1 + \ldots$

D. $6 + 9 + 13.5 + 20.25 + \ldots$

Sum of an Infinite Geometric Series

The sum of an infinite geometric series S with common ratio r and $|r| < 1$ is

$$S = \frac{a_1}{1 - r},$$

where a_1 is the first term.

Find the sum of each geometric series, if it exists.

A. $81 + 27 + 9 + 1 + \ldots$

B. $\sum_{k=1}^{\infty} 6(0.2)^k = 1.2 + 0.24 + 0.048 + \cdots$

C. $1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \cdots$

D. $\sum_{k=1}^{\infty} 4(0.5)^{k-1}$

E. $500 - 300 + 180 - 108 + \ldots$

F. $\sum_{k=1}^{\infty} \frac{1}{4}\left(\frac{4}{3}\right)^k$