Name____________________________________

Honors Algebra 2 — Ch 13 Notes Packet

Section 13.1: Special Right Triangles & Right Triangle Trigonometry
Learning Target: We are learning about special right triangles and right triangle trigonometry **Success Criteria:** • **I can understand and use trigonometric relationships of acute angles in triangles.** • **I can determine side lengths of right triangles by using trigonometric functions.**

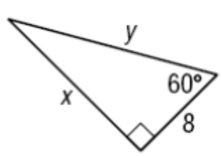
From our discovery, we saw that for the **special right triangles** (aka 45-45-90 and 30-60-90):

45-45-90 triangles:	30-60-90 triangles:
hypotentuse =	*long* = hypotentuse =

We can solve for missing sides of a triangle using these relationships.

Ex1: Solve for the missing side(s) using special triangle

A.



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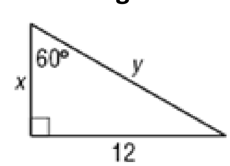

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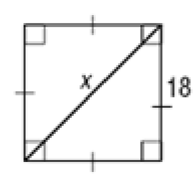

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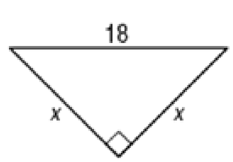


E. The perimeter of a square is 48 meters. Find the length of a diagonal.

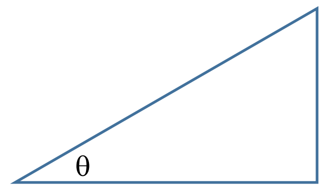
F. An equilateral triangle has a side length of 10 inches. Find the length of the triangle's altitude.

If you want to measure the height of an object that is impossible or difficult to measure directly one way we can do that is to use **TRIGONOMETRY!!** **TRIGONOMETRY** is a greek word meaning "triangle measurement."

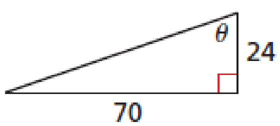
Trigonometric ratios compare ratios of 2 sides of a right triangle as related to one of the acute angles (often labeled θ, the Greek letter theta). In a right triangle that has an acute angle θ, the three sides of the right triangle are referred to as the **hypotenuse**, the side **adjacent** to θ, and the side **opposite** θ.

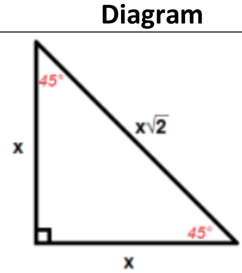
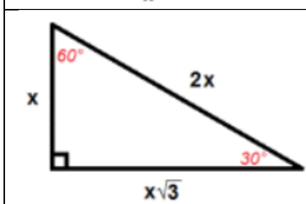
The first three trigonometric ratios are: ______________, ____________________ & ____________________

The reciprocals of the first three trigonometric ratios are:

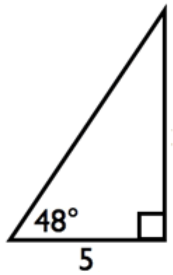


Ex2: Find the values of the six trig functions for θ

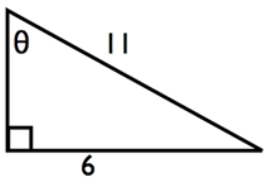


Trig Ratios of Special Right Triangles (45-45-90 & 30-60-90)			
Diagram	**Sine**	**Cosine**	**Tangent**
			
			

If you are given an acute angle of a right triangle and 1 side trig can help you find the remaining sides/ angles.



If you are given 2 sides of a right triangle, Pythagorean Theorem will help you find the 3rd side. Using trig ratios can help you find the value of the angles. In these circumstances you must UNDO the trig function to solve for the angle. When we undo a math operation we use the INVERSE operation. Trig functions also have inverses:

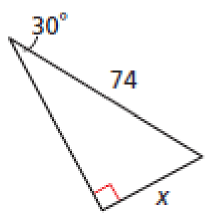


- the inverse of sine is $\sin^{-1}$
- the inverse of cosine is $\cos^{-1}$
- the inverse of tangent is $\tan^{-1}$

READING NOTATION:
"$\theta = \sin^{-1}\left(\frac{3}{5}\right)$" is read "theta equals inverse sine of 3 over 5"

Ex3: Use a trig function to find the missing sides and angles. Use special triangle formulas when possible. Round to tenths when necessary.

A.



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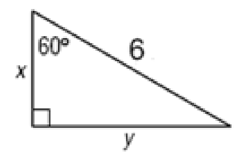

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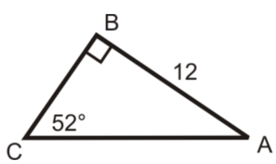

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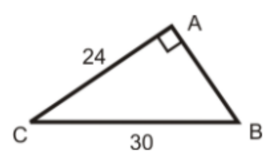
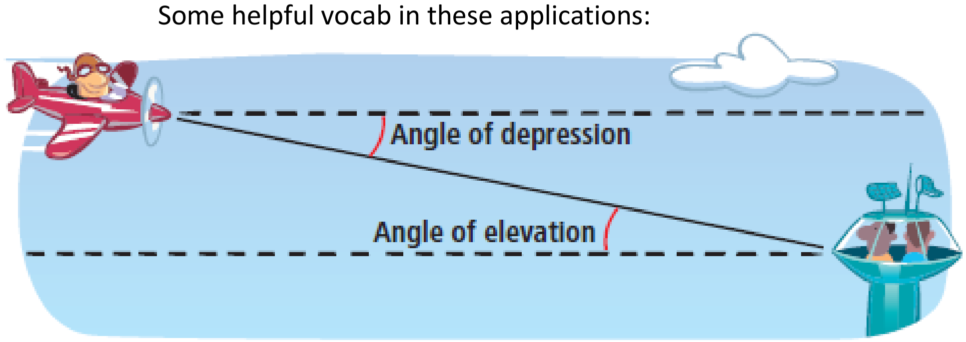


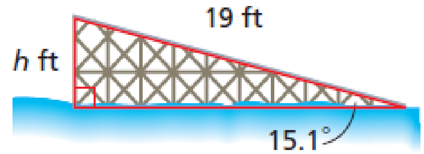
Since trigonometry was developed to solve real problems of ancient Greeks, Eqyptians and Indians, many of the math exercises we will practice often lead to some form of application to problem solving in the physical world around us. Keep your eyes open… :)

Some helpful vocab in these applications:



Ex4: Solve using trigonometry

A. In a waterskiing competition, a jump has a ramp with the measurements shown. What is the height above the water that a skier leaves the ramp?



B. A biologist is tracking tree growth. Her eye level is 6 ft above the ground and her angle of elevation to the top of the tree is 38.7°. If the biologist is standing 180 feet from the base of the tree, what is the height of the tree?

<u>Sketches! They are super helpful!</u>
↓

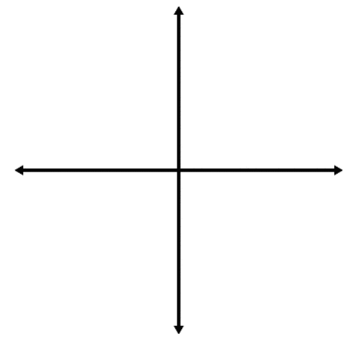
Section 13.2: Angles of Rotation
Learning Target: We are learning angles of rotation and trigonometry **Success Criteria:** • **I can draw angles in standard position.** • **I can determine the values of the trigonometric functions for an angle in standard position.**

Last lesson, we worked with acute angles of triangles. In this lesson, we will examine angles whose measures are any real number (both + and –). Since an angle is formed by 2 rays that originate at the same point (its vertex), we will look at generating angles formed by fixing one ray (the initial ray) on the positive x axis and rotating the other ray (the terminal ray) around the vertex placed at the origin.

An angle is in **STANDARD POSITION** when its vertex is at the origin and one of the rays of the angle is on the positive x-axis. **The INITIAL RAY** is the ray on the positive x-axis.

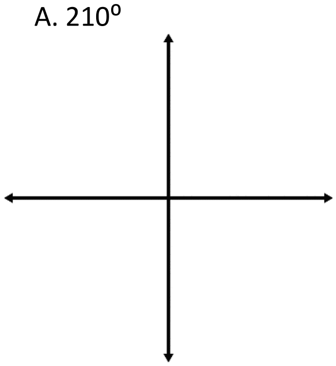
The **ANGLE OF ROTATION** is formed by fixing the position of the initial ray and changing the position of the other ray called the **TERMINAL RAY**, which is the ray that is rotated around the origin and makes up the other side of the angle. The terminal ray can be rotated any number of degrees (even more than 360 or negatively rotated).

When the terminal ray is rotated **counter clockwise**, the angle of rotation is **positive** and when the terminal ray is rotated **clockwise** , the angle of rotation is **negative**.

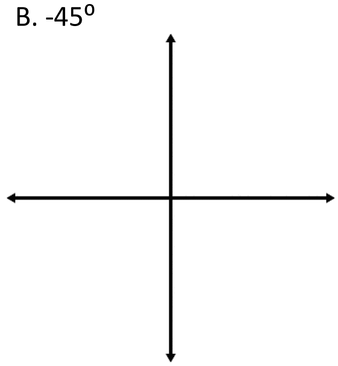


Ex1: Draw an angle with the given measure in standard position.

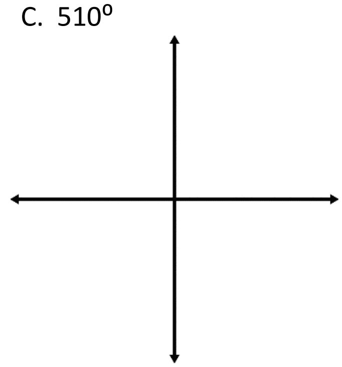
A. 210°



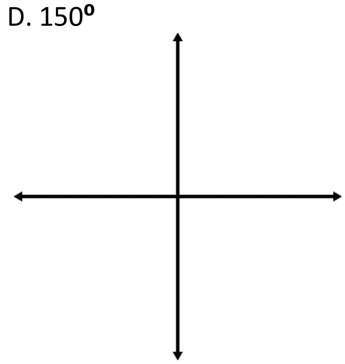
B. -45°



C. 510°



D. 150°



COTERMINAL ANGLES are angles that have the same terminal side when they are in standard position.
When an angle is in standard position a **REFERENCE ANGLE** is the positive acute angle formed by the terminal side and the x-axis.

In Ex 1, which angles are coterminal?

Ex 2: Find the measures of a positive and negative angle that is coterminal to the given angle.

A. -60°

B. 495°

For an angle in standard position, the **REFERENCE ANGLE** is the POSITIVE acute angle formed by the terminal side of the angle and the X-AXIS.

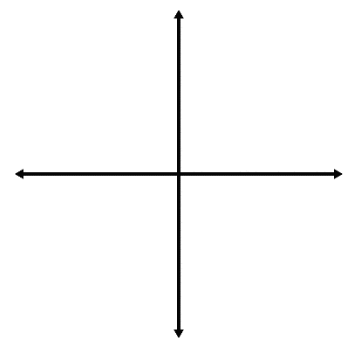
Ex 3: Find the measure of the reference angle.

A. $\theta = 135^o$

B. $\theta = -105^o$

C. $\theta = 325^o$

Suppose you have a point on the terminal side of an angle. We can visualize or sketch this and try to make connections between what we know about circles and trigonometry.



For point P(x, y) on the terminal side of θ in standard position where r =

sin θ = **cos θ =** **tan θ =**

Ex4: P(– 6, 9) is a point on the terminal side of θ in standard position. Find the exact value of the six trigonometric functions for θ.

Section 13.3: The Unit Circle

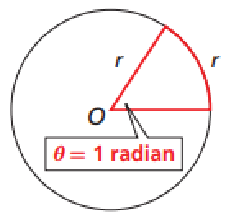
Learning Target: We are learning about the unit circle

Success Criteria:

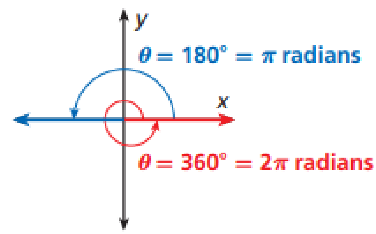
- **I can convert angle measures between degrees and radians.**
- **I can find the values of trigonometric functions on the unit circle.**

Thus far in math, angles have been measured in degrees. However, they can also be measured in radians.

- A radian is a unit of measure based on arc length.
If a central angle θ of a circle with radius r intercepts an arc of length r, then the measure of θ is define a 1 radian.



- Since the circumference of a circle is 2πr, then there are 2π radians in a circle. So a full 360^o rotation is equal to 2π radians, thus 180^o is equal to π radians.



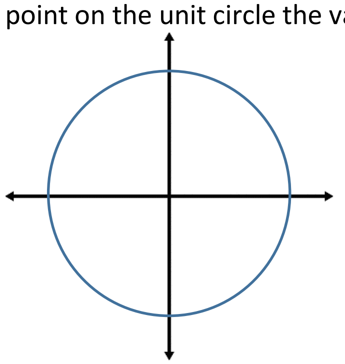
Converting Angle Measures

DEGREES TO RADIANS	RADIANS TO DEGREES
Multiply the number of degrees by $\left(\frac{\pi \text{ radians}}{180^\circ}\right)$.	Multiply the number of radians by $\left(\frac{180^\circ}{\pi \text{ radians}}\right)$.

Ex 1: Convert from degrees to radians or from radians to degrees

A. 120^o B. 35^o C. $\frac{4\pi}{3}$ D. $\frac{2\pi}{5}$

- A unit circle is a circle with a radius of 1 unit, so for every point on the unit circle the value of r = 1



⚡⚡Jump to p. 9 for Unit Circle ⚡⚡

Ex 2: Use the unit to find the exact value of:

A. $\cos 225^{o}$

B. $\sin \frac{5\pi}{6}$

C. $\tan \frac{5\pi}{6}$

D. $\tan \pi$

E. $\tan 90^{o}$

F. $\sin -330^{o}$

You can also navigate around and solve for other angles in the unit circle using **REFERENCE ANGLES**

Ex 3: Use reference angles and the unit circle to find the exact value of sin, cos and tan for:

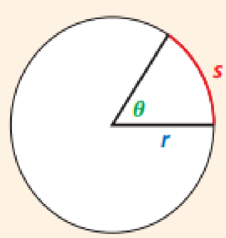
A. 150^{o}

B. $-\frac{\pi}{4}^{o}$

Arc Length Formula

For a circle of radius r, the arc length s intercepted by a central angle θ (measured in radians) is given by the following formula.

$$s = r\theta$$

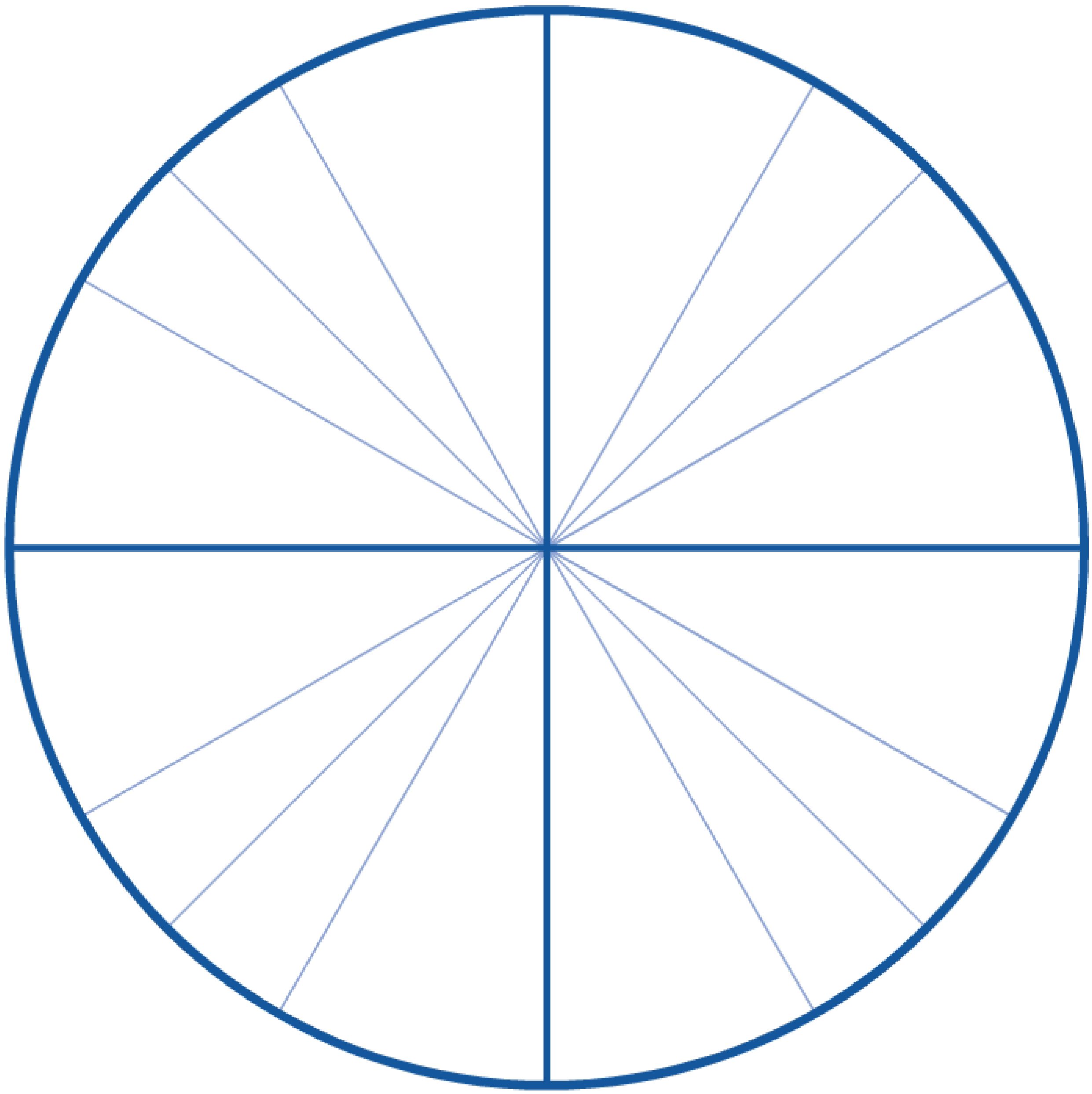


Read example 4 on p.946 for an application of arc length formula.

Ex 4: Find the arc length.

A. $r = 4$ inches and $\theta = 150^{o}$

B. $r = 2$ cm and $\theta = \frac{9\pi}{20}$



Section 13.4: Inverses of Trigonometric Functions
Learning Target: We are learning about inverses of trig functions **Success Criteria:** • **I can evaluate inverse trigonometric functions.** • **I can use trigonometric equations and inverse trigonometric functions to solve problems.**

We've seen how to undo trig functions with its inverse to solve for angles in triangles.

Trigonometric Function	Inverse Relation
$\sin\theta = a$	$\sin^{-1}a = \theta$
$\cos\theta = a$	$\cos^{-1}a = \theta$
$\tan\theta = a$	$\tan^{-1}a = \theta$

As we apply inverse trig to the values in the unit circle please note that each inverse trigonometric relation has multiple values - that's why it's a relation and not a function.

Ex 1: What angle measure produces a cosine value of $\frac{\sqrt{3}}{2}$?

Because more than one value of θ produces the same output value for a given trig function, domain restrictions are necessary for each trig function in order to define the inverse trig FUNCTION. There are many different ways you could restrict the domain, but the math community has accepted a common set of restrictions so that everyone is on the same page. When we restrict the domain of the trig functions they are indicated with a capital letter.

Inverse Trigonometric Functions

WORDS	SYMBOL	DOMAIN	RANGE
The **inverse sine function** is $\text{Sin}^{-1}a = \theta$, where $\text{Sin}\,\theta = a$.	$\text{Sin}^{-1}a$	$\{a \mid -1 \le a \le 1\}$	$\left\{\theta \mid -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}\right\}$ $\{\theta \mid -90^\circ \le \theta \le 90^\circ\}$
The **inverse cosine function** is $\text{Cos}^{-1}a = \theta$, where $\text{Cos}\,\theta = a$.	$\text{Cos}^{-1}a$	$\{a \mid -1 \le a \le 1\}$	$\{\theta \mid 0 \le \theta \le \pi\}$ $\{\theta \mid 0^\circ \le \theta \le 180^\circ\}$
The **inverse tangent function** is $\text{Tan}^{-1}a = \theta$, where $\text{Tan}\,\theta = a$.	$\text{Tan}^{-1}a$	$\{a \mid -\infty < a < \infty\}$	$\left\{\theta \mid -\frac{\pi}{2} < \theta < \frac{\pi}{2}\right\}$ $\{\theta \mid -90^\circ < \theta < 90^\circ\}$

We can use these restricted trig functions to define the inverse trig functions so that they again have the relationship of one output for every input:

Ex 2: Evaluate each inverse function. Answer in degrees and radians.

We can also use our calculators to help if we aren't given sin, cos and tan values from the unit circle.
Ex: Solve to the nearest tenth of a degree using the given restrictions.

APPLICATION:

A. A painter needs to lean a 30 ft ladder against a wall. Safety regulations recommend that the distance between the base of the ladder and the wall should be ¼ the length of the ladder. To the nearest degree, what acute angle should the ladder make with the ground?

B. The pilot of a plane wants to fly 800 miles east and 155 miles north of the airport. To the nearest degree, in what direction should he head?