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## Section 6.5: Finding Real Roots of Polynomial Equations

Learning Target: We are learning about finding all real roots of polynomial equation.

## Success Criteria:

- I can identify the multiplicity of roots.
- I can use the Rational Root Theorem and the Irrational Root Theorem to solve polynomial equations.

In finding all REAL roots of a polynomial there are many factors to consider:
$\checkmark$ Can you find roots by factoring reasonably quickly?
$\checkmark$ Are there rational roots of the polynomial?
$\checkmark$ Are there irrational roots of the polynomial?
Ex1: Solve by factoring: (would these roots be considered rational or irrational?)
A. $4 x^{6}-24 x^{5}+36 x^{4}=0$
B. $x^{4}+25=26 x^{2}$

Sometimes a polynomial has a factor that appears more than once. When this occurs it creates a multiple root. The $\qquad$ of root $r$ is the number of times $(x-r)$ is a factor of the polynomial $\mathrm{P}(\mathrm{x})$.

The multiplicity has a connection to the graph of the polynomial
$>$ When a root has an even multiplicity, the graph $\qquad$ the $x$-axis
$>$ When a root has an odd multiplicity greater than 1, the graph $\qquad$ as it crosses $x$ - axis

- Why "greater than 1?"

Ex2: Use the table function in your calculator to identify the roots. State the multiplicity of each root. Describe how the roots would appear on the graph. Graph to check.
A. $x^{3}+6 x^{2}+12 x+8=0$
B. $x^{4}+8 x^{3}+18 x^{2}-27$

It's not always easy to factor all polynomials. How can you find all possible factors?

Rational Root Theorem: If $P(x)$ has integer coefficients then every rational root of $P(x)$ can be written as $\frac{p}{q}$ where p is a factor of the constant term and q is a factor of the leading coefficient.

Ex3: Identify ALL of the possible options of roots to test for $P(x)=2 x^{3}-3 x^{2}-10 x-4$

If you've tried to find all the possible rational roots the polynomial is unfactorable, you may have IRRATIONAL ROOTS!

Irrational Root Theorem: If $P(x)$ has rational coefficients and $a+b \sqrt{ }$ is a root of $P(x)=0$, then $a-$ $b V_{c}$ is also a root of $P(x)=0$ (when $a$ and $b$ are real numbers and $V_{c}$ is irrational

Ex4: The width of a shipping crate is 2 ft less than the length, and the height is 4 ft greater than the length. What is the length if the shipping crate holds $21 \mathrm{ft}^{3}$ ?

## NO CALCULATORS

Ex5: Identify ALL of the real roots of $2 x^{3}-9 x^{2}+2=0$
[this means rational and irrational]
[Try to find rational roots first $\rightarrow$ use rational root theorem to find all possible options to test]

## YOU TRY:

Identify all real roots:
A. $x^{4}-12 x^{2}+27=0$
B. $3 x^{3}+3 x^{2}-10 x-24=0$
C. $3 x^{3}-18 x^{2}-9 x+132=0$

Section 6.6: Fundamental Theorem of Algebra
Learning Target: We are learning about finding all real roots of polynomial equation.

Success Criteria:

- I can use the Fundamental Theorem of Algebra and its corollary to write a polynomial equation of least degree with given roots.
- I can identify all of the roots of a polynomial equation.

The following statements are equivalent:

- $r$ is an $x$-intercept of the graph of $P(x)$.
- $r$ is a zero of $P(x)$.
- A real number $r$ is a root of the polynomial equation $P(x)=0$.
- $P(r)=0$.
- When you divide the rule for $P(x)$ by $(x-r)$, the remainder is 0 .
- $(x-r)$ is a factor of $P(x)$.

By knowing the zeros of a polynomial, you can create the simplest polynomial by turning the zeros into factors, multiplying them and writing them in standard form.

Ex1: Write the simplest standard form polynomial with the given zeros.
A. $-2,0,2$, and 4
B. $-1, \frac{2}{3}$ and 4

Complex Conjugate Root Theorem: If $\mathrm{a}+\mathrm{bi}$ is a root of a polynomial equation with real number coefficients then a - bi is also a root

Ex2: Write the simplest polynomial that has the zeros $-5,4 i$, and $\sqrt{ } 2$.

What have we noticed about the degree of the polynomial and its number of roots?
Fundamental Theorem of Algebra:
Every polynomial function of degree $n \geq 1$ has at least 1 zero (where a zero may be a complex number)
Corollary: Every polynomial function of degree $n \geq 1$ has exactly $n$ zeros, including multiplicities.

Ex3: Solve by finding all roots of the polynomial: $x^{4}-3 x^{3}+5 x^{2}-27 x-36=0$

Ex 4: A grain silo is in the shape of a cylinder with a hemisphere on top. The cylinder is 20 ft tall. The volume of the silo is $2106 \pi \mathrm{ft}^{3}$. Find the radius of the silo.

## YOU TRY:

1. Write the simplest standard form polynomial with the given zeros: $-5, \frac{2}{7}, 0$
2. Write the simplest polynomial that has the zeros $2,3 i$, and $\sqrt{ } 3$.
3. Solve by finding all roots of the polynomial: $x^{4}-3 x^{3}-4 x^{2}+24 x-12=0$

## Section 6.7: Graphs of Polynomials

Learning Target: We are learning about graphs of polynomial functions.

## Success Criteria:

- I can use properties of end behavior to analyze, describe, and graph polynomial functions.
- I can identify and use maxima and minima of polynomial functions to solve problems.

Graphs of polynomials have a basic shape based on their degree. Recall that the degree of every polynomial equals the number of roots/ zeros of the polynomial. The graphs below show the MAXIMUM number of times the graph of each type of polynomial can intersect the $x$-axis. If the roots have multiplicities more than 1 than the basic shape may vary.
 Degree 0


Linear function Degree 1


Quadratic function Degree 2


Cubic function Degree 3


Quartic function Degree 4


Quintic function Degree 5
 approaches positive infinity [ ] and as $x$ approaches negative infinity [

Samples:



Ex 1: Without graphing the polynomial, identify the leading coefficient and degree to determine the end behavior of the polynomial.
A. $P(x)=-x^{4}+6 x^{3}-x+9$
B. $P(x)=-2 x^{5}+6 x^{4}-x+4$

Polynomial End Behavior

| $P(x)$ has... | Odd Degree | Even Degree |
| :---: | :---: | :---: |
| Leading coefficient $a>0$ |  |  |
| Leading coefficient $a<0$ |  |  |

Ex 2: Identify whether the graphed function is a + or - leading coefficient and an even or odd degree.
A.

B.


Recall that:
$>$ When a root has an even multiplicity, the graph $\qquad$ the x -axis
> When a root has an odd multiplicity greater than 1, the graph $\qquad$ as it crosses $x$ - axis

Ex 3: Use the graph to classify the leading coefficient and minimum degree.
A.

B.


Using ALL info that can be determined from an equation, we can sketch an accurate graph of a polynomial:

## Steps to Graph a Polynomial Function:

1- Find and plot the $\qquad$ \& $\qquad$
2- Determine the $\qquad$ based on leading coefficient and degree

3- Make a $\qquad$ to find $f(x)$ output values between zeros to get the "look" of the graph

4- Sketch graph with as much accuracy as possible
Ex 4: Graph the polynomial function by hand
A. $P(x)=x^{3}+2 x^{2}-4 x-8$

B. $P(x)=x^{3}+4 x^{2}+x-6$


A $\qquad$ is where a graph changes from increasing to decreasing or from decreasing to increasing. Each turning point corresponds to a $\qquad$
$\qquad$ . "Local" is used when there is more than one max or min.

[^0]
## LOCAL MAXIMA and MINIMA (plural of maximum and minimum)

For a function $f, f(a)$ is a local maximum if there is an interval around a such that $f(x)<f(a)$ for every $x$-value in the interval except a.

For a function $\mathrm{f}, \mathrm{f}(\mathrm{a})$ is a local minimum if there is an interval around a such that $f(x)>f(a)$ for every $x$-value in the interval except a.

Ex 5: Use your graphing calculator to find an accurate estimate of the local max/mins.
A. $P(x)=2 x^{3}-18 x+1$
B. $P(x)=x^{4}-4 x^{3}+3 x+5$

Ex 6: An artist plans to construct an open box from a 15 in by 20 in sheet of metal by cutting squares from the corners and folding up the sides. Find the maximum volume of the box and the corresponding dimensions.

## YOU TRY:

1. Without graphing the polynomial, identify the leading coefficient and degree to determine the end behavior of the polynomial $P(x)=-x^{4}+6 x^{3}-x+9$.
2. Identify whether the graphed function is a + or - leading coefficient and an even or odd degree.
3. Use the graph to classify the leading coefficient and minimum degree.

4. Graph the polynomial function by hand: $P(x)=x^{4}+4 x^{3}-7 x+2$

5. Use your graphing calculator to find an accurate estimate of the local max/ mins of the polynomial $P(x)=x^{4}+4 x^{2}-6$.

## Section 6.8: Transforming Polynomials

Learning Target: We are learning about transforming the graphs of polynomial functions.

## Success Criteria:

- I can transform polynomial functions.

| Transformations of $\boldsymbol{f}(\boldsymbol{x})$ |  |  |  |
| :--- | :---: | :--- | :--- |
| Transformation | $f(x)$ Notation | Examples |  |
| Vertical translation | $f(x)+k$ | $g(x)=x^{3}+3$ 3 units up <br> $g(x)=x^{3}-4$ 4 units down |  |
| Horizontal translation | $f(x-h)$ | $g(x)=(x-2)^{3}$ 2 units right <br> $g(x)=(x+1)^{3}$ 1 unit left |  |
| Vertical stretch/compression | $a f(x)$ | $g(x)=6 x^{3}$  <br> $g(x)=\frac{1}{2} x^{3}$ stretch by 6 <br> compression by $\frac{1}{2}$  |  |
| Horizontal stretch/compression | $f\left(\frac{1}{b} x\right)$ | $g(x)=\left(\frac{1}{5} x\right)^{3}$  <br> $g(x)=(3 x)^{3}$ stretch by 5 <br> compression by $\frac{1}{3}$  |  |
| Reflection | $-f(x)$ <br> $f(-x)$ | $g(x)=-x^{3}$ <br> $g(x)=(-x)^{3}$ | across $x$-axis <br> across $y$-axis |

Ex 1: Let $f(x)=x^{3}-3$. Write the rule for each transformation, sketch the graph and describe $g$ and $h$ as a function of $f$.
A. $g(x)=f(x)+5$
B. $h(x)=f(x-2)$


Ex 2: Let $f(x)=x^{4}-8$. Write the rule only for the given transformations.
A. $g(x)=f(x-2)$
B. $g(x)=f(-x)$

Ex 3: Let $f(x)=x^{3}-2 x^{3}-x+2$. Write the rule for each transformation \& verify graphs on calculator
A. $g(x)=$ reflection across the $x$-axis
B. $h(x)=$ reflection across the $y$-axis

Ex 4: Let $f(x)=2 x^{4}-6 x^{2}+1$. Write the rule for each transformation and describe $g$ and $h$ as a function of $f$. Verify the graphs in your calculator.
A. $g(x)=\frac{1}{2} f(x)$
B. $h(x)=f\left(\frac{1}{3} x\right)$

Ex 5: Let $f(x)=6 x^{3}-3$. Write a function that transforms $f(x)$ in the given ways.
A. compress vertically by $\frac{1}{3}$ \& shift 2 units right $\quad$ B. reflect across the $y$-axis \& shift 2 units down.

Ex 6: The number of skateboards sold per month can be modeled by $f(x)=0.1 x^{3}+0.2 x^{2}+0.3 x+130$. Let $g(x)=f(x)+20$. Write the new rule for $g(x)$ and interpret/explain the meaning of the transformation in terms of monthly skateboard sales.

## YOU TRY:

1. Let $f(x)=x^{3}+1$. Write the rule for each transformation and describe $g$ and $h$ as a function of $f$.
A. $g(x)=f(x+4)$
B. $g(x)=3 f(x)$
2. Let $f(x)=x^{4}-8$. Write the rule only for $g(x)=f(x-2)$
3. Let $f(x)=-x^{3}+3 x^{2}-2 x+1$. Write the rule for transformations \& verify graphs on calculator
A. reflect across the $x$ axis
B. reflect across the $y$ axis
4. Let $f(x)=x^{3}-3$. Write a function $g(x)$ that stretches $f(x)$ vertically by a factor of 2 , reflects over the $y$-axis and shifts 5 left.

## Section 6.9: Curve Fitting with Polynomial Models

Learning Target: We are learning about using regression equations to model polynomial data.

Success Criteria:

- I can use finite differences to determine the degree of a polynomial that will fit a given set of data.
- I can use technology to find polynomial models for a given set data.


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| Finite Differences of Polynomials |  |  |
| :---: | :---: | :---: |
| Function Type | Degree | Constant Finite <br> Differences |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

Ex 1: Use finite differences to determine the degree of the polynomial.
A.

| $x$ | -6 | -3 | 0 | 3 | 6 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $y$ | -30 | 15 | 30 | 34 | 41 | 60 |

B.

| 10 | 20 | 30 | 40 | 50 | 60 | 70 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 249 | 251 | 250 | 254 | 270 | 306 | 370 |

Enter the data from Ex 1 part A into the calculator and create different regressions equations. Then do the same for part $B$. What does the $r^{2}$ value tell you?

Ex 2: Use a polynomial model to create an equation and make estimates.
A. The table shows the average price in dollars per 1000 cubic feet of natural gas for residential use in the United States from 2000 through 2010.

| Year | Price |
| :---: | :---: |
| 2000 | 3.68 |
| 2001 | 4.29 |
| 2002 | 5.17 |
| 2003 | 6.06 |
| 2004 | 6.12 |


| 2005 | 6.12 |
| :--- | :--- |
| 2006 | 5.83 |
| 2007 | 5.74 |
| 2008 | 5.71 |
| 2009 | 5.64 |
| 2010 | 5.61 |

Is a quadratic or cubic regression model a better choice? $\qquad$ Why?
B. The table shows the number of US births in thousands from 1998 to 2008

| Year | Births |
| :---: | :---: |
| 1998 | 3,942 |
| 1999 | 3,959 |
| 2000 | 4,059 |
| 2001 | 4,126 |
| 2002 | 4,092 |


| 2003 | 4,090 |
| :--- | :--- |
| 2004 | 4,112 |
| 2005 | 4,138 |
| 2006 | 4,266 |
| 2007 | 4,316 |
| 2008 | 4,248 |

Is a linear, quadratic, cubic or quartic regression model a better choice? $\qquad$ Why?

Write the best regression equation for the data. Write the best regression equation for the data.

Use the model to estimate average prices in 2013.
Use the model to estimate number of births in 1995 and 2016.

## YOU TRY:

The table to the right shows how many minutes out of each 8-hour work day are used to pay one day's worth of taxes.

1. Using your calculator, determine if a linear, quadratic or cubic regression equation best fits the data in the table. Write the equation that best fits the data.
2. How do you determine which regression equation is best?

| Year | Minutes |
| :---: | :---: |
| 1930 | 56 |
| 1940 | 86 |
| 1950 | 119 |
| 1960 | 134 |
| 1970 | 144 |
| 1980 | 147 |
| 1990 | 148 |
| 2000 | 163 |
| 2005 | 151 |

Source: Tax Foundation
3. Based on this equation, how many minutes should you expect to work each day in the year 2020 to pay one day's taxes? Explain if this is reasonable or not.


[^0]:    "Absolute" is used when there is only one.

