Honors Algebra 2 Name ____________________________________

Connecting Algebra to **Number Theory**

See Skills Bank page S70

Pascal's Triangle

Each number in Pascal's triangle is the sum of the two numbers diagonally above it. All of the outside numbers are 1.

Many interesting number patterns can be found in Pascal's triangle, such as Fibonacci's sequence and powers of 2.

Pascal's Triangle is useful for many different mathematical situations, such as expanding binomials and probability.

Row 0 → 1
Row 1 → 1 1
Row 2 → 1 2 1
Row 3 → 1 3 3 1
Row 4 → 1 4 6 4 1
Row 5 → 1 5 10 10 5 1
Row 6 →
Row 7 →

Activity

Find rows 6 and 7 of Pascal's triangle.

Row 5 → 1 5 10 10 5 1
Row 6 → 1 6 15 20 15 6 1
Row 7 → 1 7 21 35 35 21 7 1

All of the outside numbers are 1. Fill in values by adding the numbers in row 5 that are diagonally above the new values.

Repeat the process for row 7.

Try This

1. Find rows 8, 9, and 10 of Pascal's triangle.
2. **Make a Conjecture** What can you say about the relationship between the row number and the number of terms in a row?
3. **Make a Conjecture** What can you say about the relationship between the row number and the second term in each row?
4. **Make a Conjecture** Expand $(x+1)(x+1)$ *and* $(x+1)(x+1)(x+1)$, and use your answers to make a conjecture about the relationship between Pascal's triangle and the multiplication of binomials.

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Expanding a Power of a Binomial

Find the product.

$(x+y)^3$

$(x+y)(x+y)(x+y)$ *Write in expanded form.*

$(x+y)(x^2+2xy+y^2)$ *Multiply the last two binomial factors.*

$x(x^2)+x(2xy)+x(y^2)+y(x^2)+y(2xy)+y(y^2)$ *Distribute x and then y.*

$x^3+2x^2y+xy^2+x^2y+2xy^2+y^3$ *Multiply.*

$x^3+3x^2y+3xy^2+y^3$ *Combine like terms.*

Notice the coefficients of the variables in the final product of $(x+y)^3$. These coefficients are the numbers from the third row of Pascal's triangle.

Binomial Expansion		Pascal's Triangle (Coefficients)
$(a+b)^0 =$	1	1
$(a+b)^1 =$	$a+b$	1 1
$(a+b)^2 =$	$a^2+2ab+b^2$	1 2 1
$(a+b)^3 =$	$a^3+3a^2b+3ab^2+b^3$	1 3 3 1
$(a+b)^4 =$	$a^4+4a^3b+6a^2b^2+4ab^3+b^4$	1 4 6 4 1
$(a+b)^5 =$	$a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5$	1 5 10 10 5 1

Each row of Pascal's triangle gives the coefficients of the corresponding binomial expansion. The pattern in the table can be extended to apply to the expansion of any binomial of the form $(a+b)^n$, where n is a whole number.

Binomial Expansion

For a binomial expansion of the form $(a+b)^n$, the following statements are true.

1. There are $n+1$ terms.
2. The coefficients are the numbers from the nth row of Pascal's triangle.
3. The exponent of a is n in the first term, and the exponent decreases by 1 in each successive term.
4. The exponent of b is 0 in the first term, and the exponent increases by 1 in each successive term.
5. The sum of the exponents in any term is n.

Using Pascal's Triangle to Expand Binomial Expressions

Expand each expression.

A $(y-3)^4$

1 4 6 4 1 *Identify the coefficients for n = 4, or row 4.*

$[1(y)^4(-3)^0]+[4(y)^3-3^1]+[6(y)^2(-3)^2]+[4(y)^1(-3)^3]+[1(y)^0(-3)^4]$

$y^4-12y^3+54y^2-108y+81$

HW: p. 418 (31 – 34, 54, 57, 60, 62, 64, 66)

B $(4z+5)^3$

1 3 3 1 *Identify the coefficients for n = 3, or row 3.*

$[1(4z)^3(5)^0]+[3(4z)^2(5)^1]+[3(4z)^1(5)^2]+[1(4z)^0(5)^3]$

$64z^3+240z^2+300z+125$