

Determine if the sequence is geometric. If it is, find the common ratio.

1. $-1, 6, -36, 216, \dots$ Geometric
 $r = -6$

2. $-1, 1, 4, 8, \dots$ Not Geometric

3. $4, 16, 36, 64, \dots$ Not Geometric

4. $-3, -15, -75, -375, \dots$ Geometric
 $r = 5$

5. $-2, -4, -8, -16, \dots$ Geometric
 $r = 2$

6. $1, -5, 25, -125, \dots$ Geometric
 $r = -5$

The following problems include given information from geometric sequences:

Find the first five terms and the 20th term

7. $a_n = 2 \cdot \left(\frac{1}{4}\right)^{n-1}$
 $2, \frac{1}{2}, \frac{1}{8}, \frac{1}{32}, \frac{1}{128}$

8. $a_n = -4 \cdot 3^{n-1}$
 $-4, -12, -36, -108, -324$

$$a_{20} = \frac{1}{2^{38}}$$

$$a_{20} = -4,649,045,868$$

Find the common ratio, the first five terms and the explicit formula

9. $a_n = a_{n-1} \cdot 2$
 $a_1 = 2$

$$r = 2$$

$$2, 4, 8, 16, 32$$

$$a_n = 2 \cdot 2^{n-1}$$

10. $a_n = a_{n-1} \cdot -3$
 $a_1 = -3$

$$r = -3$$

$$-3, 9, -27, 81, -243$$

$$a_n = -3 \cdot (-3)^{n-1}$$

11. $a_n = a_{n-1} \cdot 3$
 $a_1 = -3$

$$r = 3$$

$$-3, 9, -27, 81, -243$$

$$a_n = -3 \cdot 3^{n-1}$$

Find the recursive formula and the next three terms in the sequence.

12. $a_1 = -4, r = 6$

$$-4, -24, -144, -864$$

$$a_n = a_{n-1} \cdot 6$$

$$a_1 = -4$$

13. $a_1 = 0.8, r = -5$

$$0.8, -4, 20, -100$$

$$a_n = a_{n-1} \cdot 5$$

$$a_1 = 0.8$$

Find the 8th term, the recursive formula and the explicit formula.

14. $a_4 = -12$ and $a_5 = -6 \quad r = \frac{1}{2}$

$a_n = a_{n-1} \cdot \frac{1}{2}$	$a_8 = -\frac{3}{4}$
$a_1 = -96$	
$a_n = -96 \cdot \left(\frac{1}{2}\right)^{n-1}$	

15. $a_5 = 768$ and $a_2 = 12$

$a_n = a_1 \cdot r^{n-1}$	$768 = a_1 (4)^{5-1}$
$\frac{768}{12} = 12 \cdot r^{5-2}$	$768 = a_1 \cdot 4^4$
$\frac{64}{12} = r^3$	$768 = a_1 \cdot 256$
$r = 4$	$\frac{64}{256} = \frac{1}{4}$
	$a_1 = 3$

$a_n = a_{n-1} \cdot 4$	
$a_1 = 3$	$a_8 = 49152$
$a_n = 3 (4)^{n-1}$	

16. $a_1 = -2$ and $a_5 = -512$

$$a_n = a_1 r^{n-1} \Rightarrow \frac{-512}{-2} = \frac{-2}{2} (r)^{5-1}$$

$$\sqrt[4]{256} = \sqrt[4]{r^4}$$

$$r = \pm 4$$

$a_n = a_{n-1} (4)$	$a_n = -2 (-4)^{n-1}$
$a_1 = -2$	or
or	$a_n = -2 (4)^{n-1}$
$a_n = a_{n-1} (-4)$	$a_8 = -32768$
$a_1 = -2$	or
	32768

17. $a_5 = 3888$ and $a_3 = 108$

$a_n = a_1 r^{n-1}$	$a_5 = a_1 \cdot r^{n-1}$
$\frac{3888}{108} = \frac{108}{108} r^{5-3}$	$\frac{3888}{1296} = \frac{1296}{1296} (b)^4$
$r^2 = 36$	$3888 = a_1 (-6)^4$
$r = \pm 6$	$a_1 = 3$

$a_n = a_{n-1} \cdot b$	$a_n = 3 \cdot (-6)^{n-1}$
or	$a_n = 3 (-6)^{n-1}$
$a_n = a_{n-1} \cdot b$	

$a_8 = 839808$
or -839808

Evaluate each geometric series.

18. $\sum_{k=1}^7 4^{k-1} = S_n = a_1 \left(\frac{1-r^n}{1-r} \right)$

$$S_7 = 1 \left(\frac{1-4^7}{1-4} \right)$$

$$= \frac{1-16384}{-3}$$

$$= -\frac{16383}{-3}$$

$S_7 = 5461$

19. $\sum_{k=1}^8 2 \cdot (-2)^{k-1} \quad a_1 = 2 (-2)^1 = 2 \cdot 2^0$

$$S_8 = 2 \left(\frac{1-(-2)^8}{1-(-2)} \right) = 2 \left(\frac{1-256}{3} \right) = 2 (-85)$$

$S_8 = -170$

20. $\sum_{n=1}^{10} 4 \cdot (-3)^{n-1} \quad a_1 = 4$

$$S_{10} = 4 \left(\frac{1 - (-3)^{10}}{1 + (-3)} \right) \\ = 4 \left(\frac{-59048}{4} \right)$$

$$\boxed{S_{10} = -59,048}$$

$\underbrace{x-s}_{r=s}$ $\underbrace{x-s}_{r=-s}$ $r = -s$

21. $2 - 10 + 50 - 250 \dots, n = 8$

$$S_8 = 2 \left(\frac{1 - (-5)^8}{1 + (-5)} \right) = \frac{2(390624)}{63}$$

$$\boxed{S_8 = -130,208}$$

22. $\underbrace{-3}_{x^2} - \underbrace{6}_{x^2} - \underbrace{12}_{x^2} - \underbrace{24}_{x^2} \dots, n = 9$

$$S_9 = -3 \left(\frac{1 - 2^9}{1 - 2} \right) = \frac{-3(-511)}{1} \quad r = 2$$

$$\boxed{S_9 = -1533}$$

24. $a_1 = 4, a_n = 1024, r = -2$

$$a_n = a_1 \cdot r^{n-1}$$

$$1024 = 4 \cdot (-2)^{n-1}$$

$$\frac{1024}{4} = \frac{4}{4} \cdot (-2)^{n-1}$$

$$256 = (-2)^{n-1}$$

$$(-2)(256) = \frac{(-2)^n}{(-2)^1} \cdot -2$$

$$-512 = -2^n$$

$$-512 = (-2)^9 = (-2)^n$$

$$n = 9$$

$$S_9 = 4 \left(\frac{1 - (-2)^9}{1 + (-2)} \right) \\ = 4 \frac{513}{3}$$

$$\boxed{S_9 = 684}$$

23. $\underbrace{1}_{x^2} - \underbrace{5}_{x^2} + \underbrace{25}_{x^2} - \underbrace{125}_{x^2} \dots, n = 7$

$$S_7 = 1 \left(\frac{1 - (-5)^7}{1 + (-5)} \right) \\ = \frac{78126}{6}$$

$$\boxed{S_7 = 13,021}$$

25. $a_1 = 4, a_n = 8748, r = 3$

$$a_n = a_1 \cdot r^{n-1}$$

$$8748 = \frac{4}{4} \cdot 3^{n-1}$$

$$2187 = 3^{n-1}$$

$$\log_3 2187 = n-1$$

$$\frac{n-1}{n-8}$$

$$S_8 = 4 \left(\frac{1 - 3^8}{1 - 3} \right) \\ = 4^2 \left(\frac{-6560}{-2} \right)$$

$$\boxed{S_8 = 13,120}$$

Determine the number of terms n in each geometric series.

26. $a_1 = -2, r = 5, S_n = -62$

$$S_n = a_1 \left(\frac{1-r^n}{1-r} \right)$$

$$\frac{-62}{-2} = \frac{-2}{-2} \left(\frac{1-5^n}{1-5} \right)$$

$$-4(-31) = \frac{1-5^n}{-4} \cdot -4$$

$$\frac{-124}{-1} = \frac{1-5^n}{-1}$$

$$\frac{-125}{-1} = \frac{-5^n}{-1}$$

$$125 = 5^n \text{ or } \log_5 125 = n$$

$$5^3 \quad \boxed{n=3}$$

28. $\sum_{m=1}^n -2 \cdot 4^{m-1} = \textcircled{42} S_n$

$$a_1 = -2 \quad r = 4$$

$$S_n = a_1 \left(\frac{1-r^n}{1-r} \right)$$

$$\frac{-42}{-2} = \frac{-2}{-2} \left(\frac{1-4^n}{1-4} \right)$$

$$-3(-21) = \frac{1-4^n}{-3} \cdot -3$$

$$\frac{-63}{-1} = \frac{1-4^n}{-1}$$

$$\frac{-64}{-1} = \frac{-4^n}{-1}$$

$$64 = 4^n$$

$$\log_4 64 = n \quad \boxed{n=3}$$

27. $a_1 = -3, r = 4, S_n = -4095$

$$S_n = a_1 \left(\frac{1-r^n}{1-r} \right)$$

$$\frac{-4095}{-3} = \frac{-3}{-3} \left(\frac{1-4^n}{1-4} \right)$$

$$-3(1365) = \frac{1-4^n}{-3} \cdot -3$$

$$\frac{-4095}{-1} = \frac{1-4^n}{-1}$$

$$\frac{-4096}{-1} = \frac{-4^n}{-1}$$

$$4^n = 4096$$

$$\log_4 4096 = \boxed{n=6}$$

29. $-4 + 16 - 64 + 256 \dots, S_n = 52428$

$$a_1 = -4 \quad r = -4$$

$$S_n = a_1 \left(\frac{1-r^n}{1-r} \right)$$

$$\frac{52428}{-4} = \frac{-4}{-4} \left(\frac{1-(-4)^n}{1-(-4)} \right)$$

$$5(-13,107) = \frac{1-(-4)^n}{5} \cdot 5$$

$$\frac{-65,536}{-1} = \frac{1-(-4)^n}{-1}$$

$$\frac{-65,536}{-1} = -\frac{(-4)^n}{-1}$$

$$65536 = (-4)^n$$

If n is odd = no solution

$$\log_4 65536 = n$$

$$\boxed{n=8}$$

So n must be even

- When n is even
(-4) base will

no longer be (-)
use \log_{14}